
ЕКОЛОГІЯ ТА РЕСУРСОЗБЕРЕЖЕННЯ

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OPTIMIZATION OF THE NANOFILTRATION PROCESS: A MINI-REVIEW

Nanofiltration is the perspective and effective process for wastewater treatment. The maintenance of optimal condition is essential for providing of effective performance of nanofiltration equipment. In this analytical review the issue of optimization of the nanofiltration process during the 2011-2020 years was considered, in particular, the topics of the setting of the optimization problem, choice of the criterion of optimality, and the formulation of the objective function with using both mechanistic and statistical models. Also, the applied methods of optimization problem solution and software were briefly considered. It was defined the statistical methods were used in a wider range in comparison with the mechanistic models based optimization, despite the limitations of the range of applicability of such an approach. Correspondingly, in most work, the specialized software for the experiment design was used. The fields of the practical application of the results of the optimization of the nanofiltration process were also analyzed. It was pointed out that for the optimization of nanofiltration, a narrower variety of approaches was applied in comparison with reverse osmosis, which is the closest to the nanofiltration process.

Keywords: purification, nanofiltration, simulation, optimization, algorithm

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Statement of the problem. Wastewater is the one of biggest environmental problem and its solution require the significant efforts especially in a case of complex pollutants. In particular, the development of effective technological systems and also the modernization and improvement of existing plants are needed [1]. In the recent years, the nanofiltration (NF) process is increasingly applied for increasing of effectivity of wastewater purification [2]. Especially, the investigations showed that tis process is effective for purification of water from micro and nanoplastics [2], organic micropollutants [3], sulfates [4] and heavy metal ions [5]. Also, microfiltration is appeared to be effective for acidic mine waters treatment [6] and food industry effluents [7-8], in particular it is possible to recover the proteins from the effluents of the potato starch production [7] and phenolic compounds from the from residual table olive fermentation brines [8]. The earlier applications of the nanofiltration in wastewater treatment is generalized in reviewing work [9]. In comparison with such widely used processes as reverse osmosis, NF is more profitable due to lower applied pressure (0.5-2 MPa), lower energy requirement, and also lower tendency to fouling [10]. The next step in the maintenance of effective performance of nanofiltration process is the providing of optimal conditions.

Analysis of previous research. As with most industrial processes, the economic and technological indicators of NF might be increased by carrying out the process in optimal conditions. The optimization can improve the performance of the existing processes without significant investments in the development of novel membranes or changes in the plant design [11]. For this reason, the interest of researchers in optimization methods is increasing [12], and the various analytical and numerical approaches to the solution of optimization problems are being developed [11].

In previous works dedicated to the simulation of pressure driven membrane process [13] and reverse osmosis [14], the issue of optimization in broad strokes was considered. However, in the limits of the review of the mathematical simulation exploring this question in detail does not seem possible. Since, the works, which generalize the development of the methods of NF process optimization, were not found and in work [15] the topic of the optimization was not represented, this work is actual.

The purpose of the current review is the systematization and generalization of approaches to the nanofiltration process. The objectives of the work include (1) review and evaluation of the range of plication of different methods of optimization of NF process; (2) determination of the practical problems, which are effectively solved using the

optimizations methods; (3) review of software, which is used for the realization of the methods the optimization of NF. To achieve this task more than 40 articles published in leading thematic journals during the 2011-2020 years were chosen. The distribution of the publication by years is shown in Fig. 1.

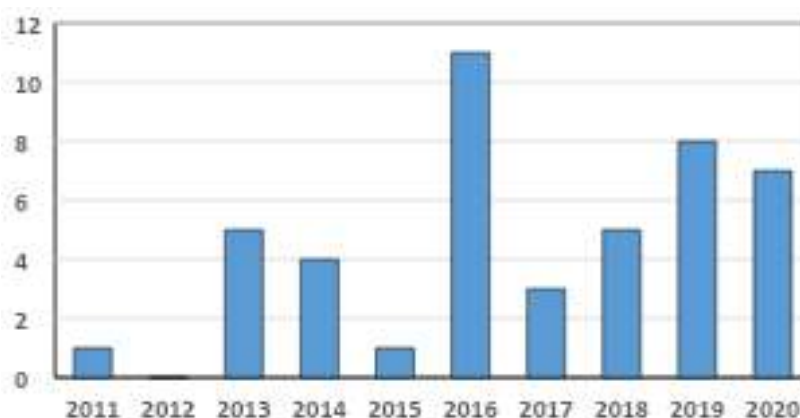


Fig. 1 – The distribution of the publication chosen for the review by years

Despite the relatively small sample size and some inhomogeneity in distribution, the trend of the increasing number of works in later years is observed.

The represented analytical review does not pretend to be comprehensive, however, it allows us to trace the modern tendencies in the optimizations of the NF process.

Setting of the optimization problem and criterion of optimality. Optimization is the process of finding the best possible solution to a certain problem by the evaluation of several alternatives [16]. Usually, optimization is based on mathematical simulation, and it is one of the most important practical applications of mathematical simulation [13-14, 16]. The optimization model includes the objective function, which is subject to maximization or minimization by the systematic choice of the variable values in the limits of the restriction imposed on it [16]. Hence, the setting of the optimization problem consists of the choice of the criterion of optimality, formulation of the objective function, and application of the restriction on it [11, 17-18].

For the optimal process design often, it is necessary to consider several goals, for example the minimal production and investment cost, the minimal energy consumption, or the maximal product quality. Moreover, these goals often contradict each other. In most cases, the optimization is carried out with a single optimality, and other parameters are introduced in the form of restrictions with fixed values [18]. In these conditions, the choice of the criterion of optimality became the key issue in the setting of the optimization problem [13-14], therefore, it is reasonable to consider in more detail, which parameters were chosen for these purposes most commonly.

In the previous reviews [13-14], it was pointed out that most commonly the economical parameters are chosen as a criterion of optimality. During the 2011–2025 years, this trend was conserved for the case of the NF process. In most cases, the minimal cost of the obtained product was chosen for these purposes, as it was realized in works [19-24]. The optimization criterion also can be formulated as the maximal difference between the concentrate cost and the treatment cost [25]. In addition, the parameters related to the issues of energy and resource-saving can be considered as economical. In this case, the criterion of optimization can be formulated as the minimal energy consumption [23, 26] and the minimization of the freshwater requirement [20].

Among the technological parameters which was used as criteria of optimality it should be noted the ones, which characterize the separation effectivity, such as maximization of removal [12, 27-30], in particular, the maximal concentration of product [31]; and also, the maximization of the product purification [21, 3] including the minimization of the COD indicator [33]. Also, the problems of the maximization of flux [12], maximization of the product yield [34], the maximization of the pressure [35], and even maximization of the area to volume ratio [36] were considered.

In work [14], it was pointed out the application of the multiobjective optimization for the choice of operation parameters of the reverse osmosis. For nanofiltration, this technique was also used in particular, in work [18] it was noted that multiobjective optimization allows the direct analysis of the impact of the contradictory parameters on the process performance and costs. However, in the same work, it is indicated that this approach is characterized by high

costs of calculations, therefore it is profitable in cases of analysis of the parameters with a high level of uncertainty, in particular for new technologies.

During the development of the objective function the mechanistic models and statistical approaches can be used, which is considered in the next section.

Approaches to optimization

Optimization based on mechanistic models. The mechanistic models are based on the ideas about physical phenomena, which take place during the occurrence of the processes, including the membrane separation process. In most cases, they need the idealizations or simplifications. The review of the main mechanistic model of the NF forces for the period from 2000 to 2010 years is represented in work [13], where the models were conditionally divided into several classes according to the based assumptions about the membrane structures and the specification of the membrane transport process. The more detailed review of such models for the period from 2011 to 2020 was made in work [15]. In the chosen materials for the current report, the models based on diffusion, pore flow, the extended Nernst-Planck equation, and computational fluid dynamics methods were considered.

The solution diffusion model is the most widely used among the diffusion based model. It was used as a basis for optimization in works [19, 21, 28]. It is based on the assumption about the dense structure of the membrane and transport through which is carried out by the sorption and diffusion of individual particles. This model appears to be suitable for the optimization of the organic solvent NF [19]. In the simplest case, the solution diffusion model describes the flux of the components through the membrane as proportional to the driving force [19]:

$$J_i = P_i \cdot \Delta DF_i \quad (1)$$

where P_i is the permeability coefficient of the component i , ΔDF_i is the driving force.

In works [21, 28], the flux equations for the particular case are described in more detail, and in work [14], for the case of reverse osmosis it is represented the detailed analysis of the parameters of the solution diffusion model, which are generally similar for the NF case. Moreover, in work [9], the solution-diffusion-imperfected model was applied. This model takes into account the additional convective transport through the pores (imperfections) in the membrane skin layer. It should be also noted that in work [21], the solution-diffusion model was complemented by the concentration polarization model.

It is necessary to note that in work [19, 21], the typical application of the solution-diffusion model for the optimization of the NF process. It consists of the formulation of the equations of this model for the considered separation problem, determination of unknown parameters of the model, design of the technique, and carrying out the experimental research for the obtaining of the numerical values of unknown parameters or correlation for calculation of this values, development the objective function on the basis of specified model equations, and defining the values of variables, which maximize or minimize the objective function.

Among the models based on the pore flow, the models based on the Darcy equation were used for the optimization of the NF process during the 2011-2020 years. According to it, the solvent flux through the membrane is determined as follows [32]:

$$J = \frac{\Delta p - \Delta \pi}{\mu_p R_m} \quad (2)$$

where Δp is the transmembrane pressure; $\Delta \pi$ is the osmotic pressure difference; μ_p is the dynamic viscosity coefficient; R_m is the resistance to membrane transport.

The using of this model for the optimization in general was the same as in the case of the solution diffusion model. This approach allows defining of the optimal flow rate of the feed solution for the given pressure [26] and the optimal design of the cascade of membrane modules [32].

In work [13, 15], it was pointed out that for the mathematical simulation of the NF process, the model based on the extended Nernst-Planck equation is the most widely used. This approach is also often applied for the optimization. The equation itself, which describes the solute flux through the membrane, can be written in a form [27]:

$$J_i = D_{i,p} \frac{dc_i}{dx} - \frac{z_i c_i D_{i,p}}{RT} F \frac{d\psi}{dx} + K_{i,c} c_i J_v \quad (3)$$

where $D_{i,p}$ is the diffusivity of the component i , c_i is the concentration of the component i , z_i is the charge of the component i , R is the gas constant; T is the absolute temperature; F is the Faraday constant; ψ is the electrical potential in membrane; $K_{i,c}$ is the permeation constant; J_v is the solute flux.

This approach to optimization was also used according to the described above scheme. It allows the solution of the problems of optimal control [27] and the optimization of the hybrid systems of purification, which include the forward osmosis process combined with NF [37].

Another approach to the optimization was based on the computational fluid dynamics methods. It consists of the numerical solution of the Navier-Stokes equation and the continuity equation adopted to the individual configuration of the membrane system. In the short form, these equations can be written in the following way [36]:

$$(\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{v} \quad (4)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (5)$$

where \mathbf{v} is the velocity vector; ∇ is the Hamilton operator; ρ is the density; p is the pressure vector; μ is the dynamic viscosity coefficient; ∇^2 is the Laplace operator.

This approach was used for the optimization of the geometrical parameters of the membrane module [36].

Optimization based on the statistical methods. Except for the mechanistic models, the statistical models, which were obtained using the methods of the design of the experiment as it was done in works [10, 12, 25, 29-31, 34, 38-46], were used quite often. The design of the experiment is the structured, organized method of the determination of relationships among the factors, which impact the process and its output characteristics. In this case, the next stage after the determination of the objective function is the determination of appropriate parameters of the process related to the sphere of investigation, which is called the influencing factors. These parameters should be controlled and can be measured with high enough accuracy. In particular, there should be the possibility to adjust and maintain the values of these parameters on the chosen level with enough precision. They also should be singular, that is not to be the function of other parameters, and concordant, which means that all possible combinations can be realized during the experiment [31].

Once the list of the process parameters under investigation is determined, it is necessary to define the possible ranges of variation of these parameters. This procedure consists of deciding the highest and lowest values in which the operating parameters are expected to vary. The choice of the range of variation is the sensible process since, if the experimental range is too narrow, only a few parameters can show the effect and this model will be limited to the narrow range. If the range is too wide, the influence of the nonlinear relationships can complicate the statistical analysis of the results. In both cases, there is a risk do not obtain the desired phenomenological understanding of the process from the investigation results [31].

Since the operating parameters are characterized by different dimensions, the conventional practice is the transformation of actual values into coded ones, as was shown in the work [31]. In this case, the factors are coded in the way in which the acquired values in range from -1 to $+1$ according to the maximal and minimum values.

Therefore, the application of the statistical methods of the design experiment allows to collect of the maximal amount of relevant information within the given required time and resources [31].

It also should be noted that the methods of the design of the experiment do not permit extrapolation. Therefore, the main limitation of the models obtained by the methods of the design of the experiment is that fact, that they are valid only in the experimental range in which they are obtained [31].

The surface response methodology (SRM) is considered to be one of the best methods of the design of the experiment. It allows the evaluation interaction of the different operating parameters with the change of response [30]. SRM allows the development of experimental models with using of appropriate mathematical and statistical techniques. In comparison with the methods based on finite volumes or computational fluid dynamics, this method is cheaper since it includes less computer simulation. However, the disadvantage of SRM is the mathematical complexity, which can appear in cases of independent responses. In these cases, the effects of the individual parameters influence the input variables, which require the introduction of coefficients that take into account the effects of interactions [45]. These specifics of the SRM led to the widest use of this approach among the statistical methods for the optimization of the NF process, as it was realized for example in works [10, 25, 39, 42, 45-46].

The response function is obtained as a result of the investigations carried out according to the methods of the design of the experiment. The simplest option is the first-order mathematical model. It can be written as follows [31]:

$$\hat{Y} = b_0 + \sum_i^k b_i X_i + \sum_i^k b_{ij} X_i X_j \quad (6)$$

where X_i, X_j are the coded values of the independent parameters; b_0, b_i, b_{ij} are the coefficients. The values of these coefficients are obtained during the statistical processing of the experimental data.

However, during the 2011-2020 years, the linear regressions according to equation (6) were used relatively rarely. In particular, except for the work [31], this model was applied in works [43] and [46]. In most works, the quadratic models were used. In this case, the regression relationship is written in a form [29]:

$$Y = \beta_0 + \sum_{j=1}^k \beta_j X_j + \sum_{j=1}^k \beta_{jj} X_{jj}^2 + \sum_{i < j=1}^k \beta_{ij} X_i X_j \quad (7)$$

In equations (6) and (7), all designations were shown as it was done in the original works. Despite to some differences, the meaning of all parameters in equation (7) is the same as in equation (6). The quadratic regression was applied in works [10, 12, 29, 34, 38, 39-42, 45].

The methods of the design of the experiment allows us to define the significance of the factors influence using the methods of the analysis of variance (ANOVA). Herewith, for these purposes, the statistical criteria are applied. In particular, the statistical significance of the correlation coefficients in equations (6) and (7) is checked using the Student's t-test, and the significance of the regression equations itself is by the F-test [40].

The relative simplicity of equations (6) and (7) allows also us to define the extremal values of such objective functions comparatively easily.

Optimization based on the economical methods. Since among the criteria of optimality, the economical parameters are most often used, in some works, the methods of economic analysis were applied for optimization [20, 22-25, 35, 45, 47-48].

In the considered works, the methods of calculation of capital (CAPEX) and operational (OPEX) costs [22, 25, 45, 48], and also the investment cost [22] were the most widely used. Furthermore, the calculations of the product cost were also used quite often [20, 25, 35, 47-48]. This has caused the application exactly the minimization of the product cost as the criterion of optimality. In some works, the technical-economical assessment [25] and shortened analysis [24] were carried out.

In these cases, the objective function was formulated based on the economic parameters in particular costs of equipment, energy, resources, depreciation charges, etc. Such functions were maximized or minimized in the same way as the objective function based on mechanistic or statistical models.

Optimal process control. As was mentioned above, for the increasing economic indices of nanofiltration equipment, it is necessary to carry out the process in optimal conditions [11]. Except for the determination of the optimal conditions, the operation of the plant in the defined condition must be provided. This issue is the subject of the optimal control [11, 17, 27, 47, 49]. In this case, it is necessary to define the objective function, which is transitional in most cases, define the parameters, which will be varied in the control of the objective function, and choose the instruments for the measurement and control of these parameters. In other words, it is necessary to implement the automatized control system [49].

During the considered period, the issue of the optimal control of the NF process was considered in the limited number of works, which were dedicated mainly to the processes of the nanofiltration in bench mode [11, 17, 47]. In addition, this approach was used for the hybrid systems [27].

Other approaches to optimization. Except for the discussed above approaches, in certain works, the issues of optimization with other methods were considered. In particular, in works [50-51], for the formulation of the objective function only the material balances were applied in work [33], the model of the boundary layer was used in work [52], the superstructure optimization was proposed, and, in work [53] the conclusion about optimal conditions was made based on the analysis of the graphical relationships obtained in the experimental way. Such approaches were used on rare occasions. However, it should be noted that, in works [51] and [52], the large objects were considered, and the nonlinear conditions were taken into account.

Algorithms and software for the solution of the optimization problems

The next step after the choice of the criterion of optimality, the development of the objective function, and imposing the restrictions is the finding of the extremal values, namely the maximums or minimums of such functions [11, 13-14, 16-18]. Since the algorithms can be quite massive, in the current work, we limit only a brief review of the most widely used algorithms, which were used for the solving of the optimization problem during the considered period. It should be noted that in some works the built-in utilities of the software were used, therefore, the most often applied program products will be briefly reviewed in this section.

It should be noted, that in some works the algorithm for solving the optimization problem was not mentioned or the calculation procedure was mentioned with reference to the previous works, as it was realized in the work [37]. As was shown above, in many cases, the minimal values of the costs, energy requirement, etc., were chosen as a criterion

of optimality. In such cases, Pontryagin's minimum principle is an effective approach to the finding of the minimal value. It is especially effective for the problems of optimal control [11, 17]. One of the methods of the realization of this approach consists in the following. The objective function should be represented as [11]:

$$\dot{\chi} = f(\chi) + g(\chi)\alpha \quad (8)$$

where χ is the state vector; α is the affinity function.

The Hamiltonian function can be written in a form [11]:

$$H(\chi, \lambda, \alpha) = 1 + f^T(\chi)\lambda + g^T(\chi)\lambda\alpha = H_0(\chi, \lambda) + H_\alpha(\chi, \lambda)\alpha \quad (9)$$

where λ is the vector of the adjacent variables which is defines as [11]:

$$\lambda = -\frac{\partial H}{\partial \chi} = -(f_\chi + g_\chi\alpha)\lambda \quad (10)$$

and [11]:

$$f_\chi(\chi) = \frac{\partial f^T(\chi)}{\partial \chi} \quad (11)$$

$$g_\chi(\chi) = \frac{\partial g^T(\chi)}{\partial \chi} \quad (12)$$

According to Pontryagin's minimum principle, the optimal solution minimizes the Hamiltonian function. Taking into account the properties of this function, the minimum is reached when α is in the following range [11]:

$$\alpha = \begin{cases} 0 & \text{if } H_\alpha(\chi, \lambda) > 0 \\ \infty & \text{if } H_\alpha(\chi, \lambda) < 0 \end{cases} \quad (13)$$

If $H_\alpha = 0$, then the Hamiltonian is singular and does not depend on α . Using this fact allows us to obtain the set of equation linear concerning λ [11]:

$$H_\alpha(\chi, \lambda) = g^T\lambda = 0 \quad (14)$$

$$\dot{H}_\alpha(\chi, \lambda) = h^T\lambda = (g_\chi f + f_\chi g)^T\lambda = 0 \quad (15)$$

$$\ddot{H}_\alpha(\chi, \lambda, \alpha) = k^T\lambda = \left(\frac{\partial h^T(\chi)}{\partial \chi} f - f_\chi h + \left(\frac{\partial h^T(\chi)}{\partial \chi} g - g_\chi h \right) \alpha \right) \lambda = 0 \quad (16)$$

This system can be solved by using the well-known mathematical methods.

In more complex cases, the methods of nonlinear programming can be used, as was realized in works [51-52]. However, in these works, a detailed description of algorithms is not provided, and the discussion is limited to the description of the applied software. The same situation is in the case of the methods of the design of the experiment. For the multi-objective optimization, in work [18], the evolution algorithm was used, but similarly without a detailed description of the calculation procedure and equations.

In general, in the case of NF, a narrower range of methods and algorithms of optimization was used in comparison with reverse osmosis, the issue of optimization of which in 2011-2020 years was briefly considered in works [14].

Among the software, first of all, the program Design Expert should be mentioned. This program was widely used for the realization of the methods of the design of experiments and analysis of variance, in particular in works [10, 12, 30, 38, 44]. It also should be noted the application of specialized program products for the analysis of the membrane equipment performance IMS Designer (Hydronautics) [50] and simulation of chemical technological processes ASPEN [18, 49]. For the more complex optimization problems, the software General Algebraic Modeling System (GAMS) was also used [20, 52]. During the application of the methods of nonlinear programming, the BARON Solver was also used [51]. In the case of computational fluid dynamics, the program product ANSYS Fluent was applied [36]. Individually, the work [37] should be noted, in which for the solving of the optimization problem, the specially developed software using the programming language Visual Basic was applied.

In general, as in the cases of optimization algorithms, the application of the specialized software appeared to be less diverse than in the case of reverse osmosis, according to the data given in the work [14].

Typical examples of the practical application of the optimization of nanofiltration process. Taking into account the important practical significance of the optimization processes, it is reasonable to consider the problems of the separation, purification, and concentration, which were realized by the NF systems, for which the optimization was carried out in the considered above publications.

First of all, it should be noted that the NF systems of the water treatment [38, 42, 46, 53, 54], in particular, potable water production [38, 42, 46], were optimized quite often. Also, a significant number of works considered the wastewater treatment, including textile [28], leather [22, 35], pharmaceutical [37] industries, productions of silk [53], olive oil [33, 49], the plants of the oil reforming [10], and also the communal wastes [23]. Moreover, in a significant number of works, the removal of the toxic components was considered [35]. In these works, the pollutants were the following: arsenic [27, 41], phosphates [29], fluorides [41], nitrates [41], Lanthanum [39], and Selenium [42]. In other works, it was considered the following problems the removal of rare earth elements [25], organic compounds [18, 21], products of hydrolysis of proteins [20], amoxicillin [12], antibiotics [43], and dyes [30]. On the other hand, the question of the concentration was also considered, in particular for the case of peptides [31].

It should be also mentioned the application of optimization for the organic solvent nanofiltration [19, 24, 51] and dnanofiltration [11, 17, 31, 47-48].

Such a wide range of applications of the optimization methods proves the significant importance of this approach to the choice of the operating parameters of the NF process.

Conclusions. Nanofiltration is an important membrane separation process that has a wide range of practical applications. To achieve the most effective operating parameters, it is necessary to provide its operation in optimal conditions. During the 2011-2020 years, more than 40 articles dedicated to the issue of NF optimization were published in the leading scientific journals. Analysis of these publications shows that the economical parameters, first of all, the cost minimization, were chosen as a criterion of optimality in most cases, whereas the technological parameters in this role were more diverse. The optimization based on the statistical methods, in particular on the methods of the design of the experiment, was applied more often than methods based on the mechanistic models. This is due to the relative simplicity of such models. However, they have a narrower range of applications.

At the same time, it should be noted that in most works dedicated to the optimization of NF, the description of the algorithm of the finding of the optimum is quite brief, and a limited number of algorithms and program products were applied. However, the methods of optimization were applied almost to the full range of the practical applications of this process.

Prospects for further research. In the future, the number of the practical applications of the nanofiltration process will be increased, and the need in providing of the optimal process condition will remain actual. Moreover, the development of the computer technologies probably will be promoting the progress in special software for the solving of the optimization problems of the nanofiltration process. In particular, the appearance the open program packages is possible.

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ОПТИМІЗАЦІЯ ПРОЦЕСУ НАНОФІЛЬТРАЦІЇ: МІНІ-ОГЛЯД

Нанofільтрація широко використовується в системах очищення стічних вод і технологічних процесах, а сфера використання цієї технології постійно розширюється. Для забезпечення найбільш ефективної роботи необхідно забезпечити оптимальну роботу процесу, проте існує декілька різноманітних підходів до оптимізації нанofільтрації. Поточна робота є спробою систематизувати та узагальнити ці підходи на основі сучасних публікацій.

Першим і визначальним кроком в оптимізації є постановка задачі оптимізації та вибір критерію оптимальності. В більшості розглянутих досліджень використовувалися економічні критерії оптимальності, в першу чергу мінімізації вартості отриманого продукту. Також в ряді робіт розглядався широкий спектр критеріїв оптимальності на основі технологічних параметрів. В той же час багатокритеріальна оптимізація використовувалася лише в окремих дослідженнях.

Цільова функція в більшості випадків розроблялася на основі механістичних моделей та статистичних методів. Найбільш використовувані механістичні моделі ґрунтувалися на розширеному рівнянні Нернста-Планка, моделей на основі дифузії та потоку в порах, а також методах обчислювальної гідродинаміки. Серед статистичних методів переважали методи планування експерименту, зокрема методологія поверхні відгуку та метод аналізу змінних. Варто відзначити, що в більшості робіт використовувалася квадратична функція відгуку. Також використовувалися методи на основі економічного аналізу та теорії автоматичного керування.

Для розв'язку задачі оптимізації в більшості відносно простих випадків використовувався метод мінімізації Понтрягіна. Для більш складних випадків використовувалися методи нелінійного програмування, а для випадку багатокритеріальної оптимізації – еволюційний алгоритм.

В більшості випадків оптимізацію систем нанofільтрації проводили для випадку використання процесу в системах очищення стічних вод окремих виробництв чи окремих шкідливих домішок. Також в значній кількості робіт розглядалися процеси підготовки питної води.

Загалом даний міні-огляд систематизує та узагальнює підходи до оптимізації процесу нанofільтрації та може бути використаний як підґрунтя для вибору стратегії оптимізації цього процесу при розробці нових установок та систем.

Ключові слова: очищення, нанofільтрація, моделювання, оптимізація, алгоритм

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