# АВТОМАТИЗАЦІЯ ТЕХНОЛОГІЧНИХ ПРОЦЕСІВ

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## DEVELOPMENT OF A SPRAY DRYER'S MATHEMATICAL MODEL FOR CONTROL TASKS

*The sustainable development paradigm includes the most efficient material and energy resources using in production processes. One of the ways to achieve this goal is creating the effective automated control systems. At the same time, such systems effective functioning is impossible without adequate mathematical models for control objects. Thus, the actual task of this study is to create the control object – a spray dryer -model-, which could be used in the control system. It was established that one of the main devices in the sodium tripolyphosphate production is a spray dryer in which the aqueous salts suspension is dried by flue gases. The obtained powdery product which is fed into the calcination furnace. The work investigated convective drying, where the material is in direct contact with the drying agent – furnace gas. To drying process control by the drying agent temperature.*

*The existing works analysis showed that the mathematical model developing process of a spray dryer requires in-depth research into the physical nature of processes which taking object's operation's various factors into account. The object presentation can be carried out with different approaches taking into account various important production factors, but at the same time it should be as close as possible to the nature of process. The development of the spray drying process model is carried out for regulation the moisture at the finished product.*

*The beginning of the research is the structural and parametric diagram creation of a spray dryer taking into account all input and output values which allow to determine the control disturbance channels. A material balance based on the moisture and loose matter amount and a gas environment heat balance is compiled. The transfer functions component calculation is carried out using the Kramer method.*

*The presented results of the research make it possible to build and analyse the spray dryer mathematical model taking into account the requirements for the substance moisture content be dried and the control process analysis by the drying agent temperature changing. The transient characteristics of the spray dryer by the disturbance and control channels, which are calculated and presented in the research make it possible to compare the control object behaviour with and without taking heat loss into the environment the assumptions into account.*

***Keywords:*** *energy efficiency, spray dryer, mathematical model, canonical form, linearized equation.*

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**Introduction**. Among the large number of technological processes, which lead to obtaining particulate materials, the spray drying is frequently used. The most significant of its advantages are simplicity, ease in operating, the ability to produce in one step and feasibility [1]. That allow to use this process in a wide variety of industries, for example predominantly food and pharmaceutical. In another hand, both modelling and automation control of such processes still are big challenge. Some popular methods, which widely used for control and modelling of industrial processes could hardly be applied for spray drying. The spray dryer is a complex device, which mathematical modelling should take into account many different operation factors and internal processes [2]. The research for development the mathematical model of spray dryer is necessary to take into account the main aspects of the spray drying process, and the physical principles underlying the of this device operation. The main trend in the spray dryers research and development is the material and energy costs reduction for the drying process and device quality control. Thus, the parameters of controlling the heating agent consumption and controlling its temperature to prevent the material removal and overheating are investigated. [3].

**Literature analysis and problem statement**. The general aspects of drying processes and modelling of them are presented at [4]. In industry, the polyphosphates are obtained by neutralizing phosphoric acid with alkali or soda. First, the phosphoric acid is neutralized with soda to form a solution in which the $Na_2O$ to $P_2O_5$ ratio is 5:3. The product is dried and dehydrated in rotary ovens at a temperature of 250 – 400 °C. The mixture is dried by combustion gases to 300 – 400 °C [4]. Baked particles, the size of which is up to 10 mm, are cooled and crushed. The aqueous salts suspension is dried in a spray dryer and a powdery product is obtained, which is then move to the calcination furnace.

The development of the spray dryer's mathematical model requires in-depth research into the nature of the physical process, taking into account various factors in the operation of the device. Important aspects analysis of such modelling is carried out in the [5]. It is shown that the results research on spray towers hydrodynamics detected dimensional flow patterns of gas and dispersed phase and fluctuations in size of the recirculation zones. There were made a lot of different attempts for determination agglomeration process mechanism, but none of the particles agglomeration models were validated. But this work resumed that the progress in flow hydrodynamics and agglomeration mechanism modelling allow to develop the drying kinetics determination methods, which should be an important steps towards reliable drying process research.

Similarly, in [6], the development of mathematical model takes into account the simplifications using the modern methods of experimental research, while the research in [6] is carried out from the point of the introducing drying product possibility, due to which control is used by changing the temperature of the drying agent. Also, the simulation takes into account the contact between the device and environment, which is omitted by the authors of [7].

A number of works dedicated to the mathematical modelling of individual salinities of the specified process should be noted separately. For example, the work [8] is dedicated to the modelling of the process of alumina drying at spray dryers. The this work established that with temperature and humidity changes in gas chamber dryer, changes in moisture or gas and increases in air flow, due to droplets moisture evaporation and enter the moisture to gas observed and gas temperature decreasing. It could also due to required energy supplied for evaporation of droplets moisture in gas flow. The paper [9] is dedicated to the modelling of computational fluid dynamics of air flow. The mathematical models, proposed by authors, are complicated enough, but could be successfully used in different spray dryer geometries. The results of the research showed that velocity had both positive and negative effects on the deposition rate on the spray dryer walls.

In the paper [10], development of model for a mine dryer is discussed. The author's approach to the created model consisted in reducing the dynamic values as constant, for the created flexible model, thus the moisture content of the drying agent is simplified.

The deep analysis the large number of paper about mathematical modelling of spray drying processes are carried out in [11]. The models introduced in these works are an essential step for the development of mathematical modelling of spray drying processes. However, their use in automated control systems is complicated due to the specificity of the methods used by the authors. The mathematical model developed in this research is aimed at solving the researched process automatic control problems and takes into account:

- boundary conditions, which are expressed in the flow parameters at the entrance to and exit from the apparatus, such as temperature, fuel gas humidity, fuel gas speed of, dried material characteristics;
- physical processes occurring in the dryer, both mass exchange and heat exchange;
- dryer geometry, the drying chamber shape and size, the disperser type and size,
- sprayed material particle size.

**The purpose** of the article is to develop the mathematical model of a spray dryer taking into account the requirements for the moisture content of the dried substance and to analyse the control process of the drying agent temperature change.

**Presenting main material.** One of the main apparatus in the sodium tripolyphosphate production, is a spray dryer. It this apparatus an aqueous salts suspension is dried with the help of flue gases at a temperature 320 – 480 °C and a powdery product is obtained [4] The calculation scheme of the spray dryer is shown at figure 1.

Figure 1 shows the following parameters: $G_{c.1}$ – suspension consumption; $w_{c.1}$ – suspension relative moisture content; $T_{c.1}$ – suspension temperature; $c_{c.}$ – specific suspension heat capacity; $G_{c.2}$ – dry matter consumption; $w_{c.2}$ – dry matter relative moisture content; $T_{c.2}$ – dry matter temperature; $G_{g.}$ – fuel gas consumption; $w_{g.1}$ – input fuel gas relative moisture content; $T_{g.1}$ – input fuel gas temperature; $c_g$ – fuel gas heat capacity; $w_{g.2}$ – output fuel gas relative moisture content; $T_{g.1}$ – output fuel gas temperature; $V_g$ – the volume filled with fuel gas; $\rho_g$ – fuel gas density; $V_c$ – the dryer volume occupied by the suspension (material to be dried); $\rho_c$ –suspension density; $K_T$ – heat transfer coefficient; $S$ – the heat exchange surface area between the suspension and the combustion gases; $r$ – specific water vapour formation heat capacity; $K_{T1}$ – heat transfer coefficient to the environment; $S_1$ – the heat exchange surface area between the combustion gases and the environment; $T_{Env}$ – environmental temperature.

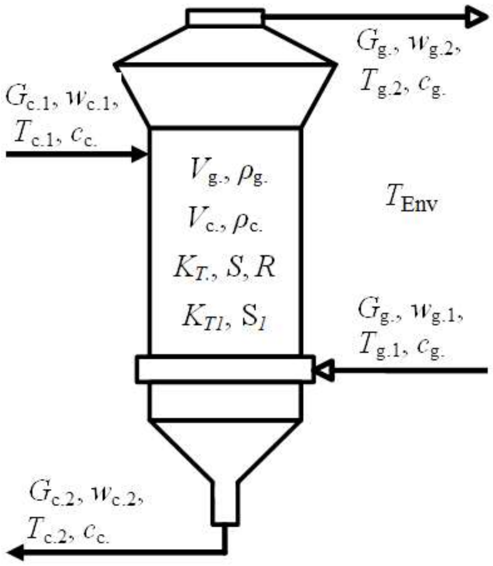


**Fig. 1 – Calculation scheme of the spray dryer [4]**

At the research the convective drying process is studied, where the material is in direct contact with the drying agent, in our case, combustion gas. In the process, the flue gas transfers its heat to the material and cools as it's moisture content increases. To control the drying process, the consumption of the drying agent is controlled, or its temperature, which is more expensive compared to the first option. In our case, we considered the option in which an increase in the consumption of fuel gas can cause the introduction of material. Therefore, the process is controlled by changing the temperature of the drying agent.

Control of the drying process is carried out either by changing the drying agent flow rate or by changing its temperature. The last option is economically less profitable, but it's used in the study. After all, a change in the drying agent flow rate can lead to removal the material. To simplify the model, we accept the following assumptions:

- the relative moisture content of the suspension at the entrance to the dryer, the relative moisture content of fuel gas at the input, and the temperature of the suspension at the entrance to the dryer are constant $w_{c.1}$ = const, $w_{g.1}$ = const, $T_{c.1}$ = const;
- the dryer is an apparatus with a linear distribution along the height the following parameters: the dried substance humidity, the flue gases humidity and temperature and the dried material temperature;
- the moisture drying process takes place from the surface of the material into the gas environment and is decisive for this process speed;
- neglecting the change in flue gas consumption due to the increase in humidity;
- the heat from the combustion gases went not only to evaporation, but also to the heating the dry substance from $T_{c.1}$ to $T_{c.2}$, and the wet substance from $T_{c.1}$ to $T_{g.2}$.

The development of mathematical models in this study is based on two different additional assumptions. In the first one, assumed that there is no heat loss to the environment. In the second – that the heat loss to the environment is proportional to the temperature difference between the combustion gas in the dryer and the external environment and is equal to $K_{T1.}S_1\left(\frac{T_{g.1}+T_{g.2}}{2}-T_{Env}\right)$.

The main flows for which the balance equations are compiled are suspension and fuel gas. Material balance according to the moisture of the bulk substance amount:

$$G_{c.1}w_{c.1}-G_{c.2}w_{c.2}-\frac{K_{T.}S}{r}\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{c.}\rho_{c.}\frac{d}{dt}\left(\frac{w_{c.1}+w_{c.2}}{2}\right)$$

Material balance according to the moisture in the fuel gas amount:

$$G_{g.}w_{g.1}-G_{g.}w_{g.2}+\frac{K_{T.}S}{r}\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{g.}\rho_{g.}\frac{d}{dt}\left(\frac{w_{g.1}+w_{g.2}}{2}\right)$$

Heat balance for the dried material:

$$G_{c.1}c_{c.}T_{c.1}-G_{c.2}c_{c.}T_{c.2}+K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{c.}\rho_{c.}c_{c.}\frac{d}{dt}\left(\frac{T_{c.1}+T_{c.2}}{2}\right)$$

Heat balance for the fuel gas:

$$G_{g.}c_{g.}T_{g.1}-G_{g.}c_{g.}T_{g.2}-K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{g.}\rho_{g.}c_{g.}\frac{d}{dt}\left(\frac{T_{g.1}+T_{g.2}}{2}\right)$$

Heat balance for the drying gas taking into account the heat loss to the environment:

$$G_{g.}c_{g.}T_{g.1}-G_{g.}c_{g.}T_{g.2}-K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)-K_{T1.}S_{1}\left(\frac{T_{g.1}+T_{g.2}}{2}-T_{Env}\right)=V_{g.}\rho_{g.}c_{g.}\frac{d}{dt}\left(\frac{T_{g.1}+T_{g.2}}{2}\right)$$

Thus, to develop a mathematical model for automated control tasks, we accept:

- state parameters: dried substance temperature $T_{c.2}$; combustion gas at the outlet temperature $T_{g.2}$; relative moisture content of fuel gas at the output $w_{g.2}$; relative moisture content of dry matter $w_{c.2}$ (controlled value);
- control action: temperature of fuel gas at the entrance $T_{g.1}$;
- disturbance: suspension flow rate $G_{c.1}$;
- variables to be linearized: $G_{c.1}$, $T_{c.2}$, $T_{g.1}$, $T_{g.2}$, $w_{c.2}$, $w_{g.2}$.

To obtain equations in linearized deviations [12], take:

$$G_{c.1}w_{c.1}-G_{c.2}w_{c.2}-\frac{K_{T.}S}{r}\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{c.}\rho_{c.}\frac{d}{dt}\left(\frac{w_{c.1}+w_{c.2}}{2}\right)$$

Build similar:

$$\Delta G_{c.1}\left(2w_{c.1}-2w_{c.2}\right)-\Delta w_{c.2}\left(2G_{c.2}\right)-\Delta T_{g.1}\left(\frac{K_{T.}S}{r}\right)-\Delta T_{g.2}\left(\frac{K_{T.}S}{r}\right)+\Delta T_{c.2}\left(\frac{K_{T.}S}{r}\right)=V_{c.}\rho_{c.}\frac{d(\Delta w_{c.2})}{dt} \quad (1)$$

Reduce to the canonical form and adopt the following notations:

$$T_{w_{c.2}}=\frac{V_{c.}\rho_{c.}}{2G_{c.2}}$$

$$K_{T_{g.2}/w_{c.2}}=\frac{K_{T.}S}{2rG_{c.2}}$$

$$K_{T_{c.2}/w_{c.2}}=\frac{K_{T.}S}{2rG_{c.2}}$$

$$K_{G_{c.1}/w_{c.2}}=\frac{w_{c.1}-w_{c.2}}{G_{c.2}}$$

$$K_{T_{g.1}/w_{c.2}}=\frac{K_{T.}S}{2rG_{c.2}}$$

Substitute these notations in (1) and obtain a linearized equation in deviations in the canonical form:

$$T_{w_{c.2}}\frac{d(\Delta w_{c.2})}{dt}+\Delta w_{c.2}+K_{T_{g.2}/w_{c.2}}\Delta T_{g.2}-K_{T_{c.2}/w_{c.2}}\Delta T_{c.2}=K_{G_{c.1}/w_{c.2}}\Delta G_{c.1}-K_{T_{g.1}/w_{c.2}}\Delta T_{g.1}$$

Perform the Laplace transformation:

$$T_{w_{c.2}}p\cdot w_{c.2}(p)+w_{c.2}(p)+K_{T_{g.2}/w_{c.2}}T_{g.2}(p)-K_{T_{c.2}/w_{c.2}}T_{c.2}(p)=K_{G_{c.1}/w_{c.2}}G_{c.1}(p)-K_{T_{g.1}/w_{c.2}}T_{g.1}(p)$$

Sum the similar terms and get the fourth equation in the variables:

$$\left(T_{w_{c.2}}p+1\right)w_{c.2}(p)+K_{T_{g.2}/w_{c.2}}T_{g.2}(p)-K_{T_{c.2}/w_{c.2}}T_{c.2}(p)=K_{G_{c.1}/w_{c.2}}G_{c.1}(p)-K_{T_{g.1}/w_{c.2}}T_{g.1}(p) \quad (2)$$

Consider the second equation:

$$G_{c.1}c_{c.}T_{c.1}-G_{c.2}c_{c.}T_{c.2}+K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{c.}\rho_{c.}c_{c.}\frac{d}{dt}\left(\frac{T_{c.1}+T_{c.2}}{2}\right)$$

Build similar:

$$-\Delta w_{g.2}\left(2G_{g.}\right)+\Delta T_{g.1}\left(\frac{K_{T.}S}{r}\right)+\Delta T_{g.2}\left(\frac{K_{T.}S}{r}\right)-\Delta T_{c.2}\left(\frac{K_{T.}S}{r}\right)=V_{g.}\rho_{g.}\frac{d(\Delta w_{g.2})}{dt} \quad (3)$$

Reduce to the canonical form and adopt the following notations:

$$T_{w_{g.2}}=\frac{V_{g.}\rho_{g.}}{2G_{g.}}$$

$$K_{T_{g.2}/w_{g.2}}=\frac{K_{T.}S}{2rG_{g.}}$$

$$K_{T_{c.2}/w_{g.2}}=\frac{K_{T.}S}{2rG_{g.}}$$

$$K_{T_{g.1}/w_{g.2}}=\frac{K_{T.}S}{2rG_{g.}}$$

Substitute these notations in (3) and obtain a linearized equation in deviations in the canonical form:

$$T_{w_{g.2}}\frac{d(\Delta w_{g.2})}{dt}+\Delta w_{g.2}-K_{T_{g.2}/w_{g.2}}\Delta T_{g.2}+K_{T_{c.2}/w_{g.2}}\Delta T_{c.2}=K_{T_{g.1}/w_{g.2}}\Delta T_{g.1}$$

Perform the Laplace transformation:

$$T_{w_{g.2}}p\cdot w_{g.2}(p)+w_{g.2}(p)-K_{T_{g.2}/w_{g.2}}T_{g.2}(p)+K_{T_{c.2}/w_{g.2}}T_{c.2}(p)=K_{T_{g.1}/w_{g.2}}T_{g.1}(p)$$

Sum the similar terms and get the fourth equation in the variables:

$$\left(T_{w_{g.2}}p+1\right)w_{g.2}(p)-K_{T_{g.2}/w_{g.2}}T_{g.2}(p)+K_{T_{c.2}/w_{g.2}}T_{c.2}(p)=K_{T_{g.1}/w_{g.2}}T_{g.1}(p) \quad (4)$$

Consider the third equation:

$$G_{c.1}c_{c.}T_{c.1}-G_{c.2}c_{c.}T_{c.2}+K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{c.}\rho_{c.}c_{c.}\frac{d}{dt}\left(\frac{T_{c.1}+T_{c.2}}{2}\right)$$

Build similar:

$$\Delta G_{c.1}\left(2c_{c.}T_{c.1}-2c_{c.}T_{c.2}\right)-\Delta T_{c.2}\left(2G_{c.2}c_{c.}+K_{T.}S\right)+\Delta T_{g.1}\left(K_{T.}S\right)+\Delta T_{g.2}\left(K_{T.}S\right)=V_{c.}\rho_{c.}c_{c.}\frac{d(\Delta T_{c.2})}{dt} \quad (5)$$

Reduce to the canonical form and adopt the following notations:

$$T_{T_{c.2}}=\frac{V_{c.}\rho_{c.}c_{c.}}{2G_{c.2}c_{c.}+K_{T.}S}$$

$$K_{T_{g.2}/T_{c.2}}=\frac{K_{T.}S}{2G_{c.2}c_{c.}+K_{T.}S}$$

$$K_{G_{c.1}/T_{c.2}}=\frac{2c_{c.}\left(T_{c.1}-T_{c.2}\right)}{2G_{c.2}c_{c.}+K_{T.}S}$$

$$K_{T_{g.1}/T_{c.2}}=\frac{K_{T.}S}{2G_{c.2}c_{c.}+K_{T.}S}$$

Substitute these notations in (5) and obtain a linearized equation in deviations in the canonical form:

$$T_{T_{c.2}}\frac{d(\Delta T_{c.2})}{dt}+\Delta T_{c.2}-K_{T_{g.2}/T_{c.2}}\Delta T_{g.2}=K_{G_{c.1}/T_{c.2}}\Delta G_{c.1}+K_{T_{g.1}/T_{c.2}}\Delta T_{g.1}$$

Perform the Laplace transformation:

$$T_{T_{c.2}}p\cdot T_{c.2}(p)+T_{c.2}(p)-K_{T_{g.2}/T_{c.2}}T_{g.2}(p)=K_{G_{c.1}/T_{c.2}}G_{c.1}(p)+K_{T_{g.1}/T_{c.2}}T_{g.1}(p)$$

Build similar:

$$\left(T_{T_{c.2}}p+1\right)T_{c.2}(p)-K_{T_{g.2}/T_{c.2}}T_{g.2}(p)=K_{G_{c.1}/T_{c.2}}G_{c.1}(p)+K_{T_{g.1}/T_{c.2}}T_{g.1}(p) \tag{6}$$

Consider the fourth equation:

$$G_{g.}c_{g.}T_{g.1}-G_{g.}c_{g.}T_{g.2}-K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)=V_{g.}\rho_{g.}c_{g.}\frac{d}{dt}\left(\frac{T_{g.1}+T_{g.2}}{2}\right)$$

Build similar:

$$\Delta T_{g.1}\left(2G_{g.}c_{g.}-K_{T.}S\right)-\Delta T_{g.2}\left(2G_{g.}c_{g.}+K_{T.}S\right)+\Delta T_{c.2}\left(K_{T.}S\right)=V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.1}\right)}{dt}+V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.2}\right)}{dt} \tag{7}$$

Reduce to the canonical form and adopt the following notations:

$$T_{T_{g.2}}=\frac{V_{g.}\rho_{g.}c_{g..}}{2G_{g.}c_{g.}+K_{T.}S}$$

$$K_{T_{c.2}/T_{g.2}}=\frac{K_{T.}S}{2G_{g.}c_{g.}+K_{T.}S}$$

$$K_{T_{g.1}/T_{g.2}}=\frac{2G_{g.}c_{g.}-K_{T.}S}{2G_{g.}c_{g.}+K_{T.}S}$$

Substitute these notations in (7) and obtain a linearized equation in deviations in the canonical form:

$$T_{T_{g.2}}\frac{d\left(\Delta T_{g.2}\right)}{dt}+\Delta T_{g.2}-K_{T_{c.2}/T_{g.2}}\Delta T_{c.2}=K_{T_{g.1}/T_{g.2}}\Delta T_{g.1}-T_{T_{g.2}}\frac{d\left(\Delta T_{g.1}\right)}{dt}$$

Perform the Laplace transformation:

$$T_{T_{g.2}}p\cdot T_{g.2}(p)+T_{g.2}(p)-K_{T_{c.2}/T_{g.2}}T_{c.2}(p)=K_{T_{g.1}/T_{g.2}}T_{g.1}(p)-T_{T_{g.2}}p\cdot T_{g.1}(p)$$

Build similar:

$$\left(T_{T_{g.2}}p+1\right)T_{g.2}(p)-K_{T_{c.2}/T_{g.2}}T_{c.2}(p)=\left(K_{T_{g.1}/T_{g.2}}-T_{T_{g.2}}p\right)T_{g.1}(p) \tag{8}$$

Equation (5) with taking into account heat loss to the environment.

$$G_{g.}c_{g.}T_{g.1}-G_{g.}c_{g.}T_{g.2}-K_{T.}S\left(\frac{T_{g.1}+T_{g.2}}{2}-\frac{T_{c.1}+T_{c.2}}{2}\right)-K_{T1.}S_1\left(\frac{T_{g.1}+T_{g.2}}{2}-T_{Env}\right)=$$
$$=V_{g.}\rho_{g.}c_{g.}\frac{d}{dt}\left(\frac{T_{g.1}+T_{g.2}}{2}\right)$$

Deviations of variables in the static mode are entered in parentheses:

$$\Delta T_{g.1}\left(2G_{g.}c_{g.}-K_{T.}S-K_{T1.}S_1\right)-\Delta T_{g.2}\left(2G_{g.}c_{g.}+K_{T.}S-K_{T1.}S_1\right)+\Delta T_{c.2}\left(K_{T.}S\right)=$$
$$=V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.1}\right)}{dt}+V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.2}\right)}{dt}$$

Build similar:

$$V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.2}\right)}{dt}+\Delta T_{g.2}\left(2G_{g.}c_{g.}+K_{T.}S-K_{T1.}S_1\right)-\Delta T_{c.2}\left(K_{T.}S\right)=$$
$$=\Delta T_{g.1}\left(2G_{g.}c_{g.}-K_{T.}S-K_{T1.}S_1\right)-V_{g.}\rho_{g.}c_{g.}\frac{d\left(\Delta T_{g.1}\right)}{dt} \tag{9}$$

Reduce to the canonical form and adopt the following notations:

$$T_{T_{g.2}D}=\frac{V_{g.}\rho_{g.}c_{g..}}{2G_{g.}c_{g.}+K_{T.}S-K_{T1.}S_1}$$

$$K_{T_{c.2}/T_{g.2}D}=\frac{K_{T.}S}{2G_{g.}c_{g.}+K_{T.}S-K_{T1.}S_1}$$

$$K_{T_{g.1}/T_{g.2}D}=\frac{2G_{g.}c_{g.}-K_{T.}S}{2G_{g.}c_{g.}+K_{T.}S-K_{T1.}S_1}$$

Substitute these notations in (9) and obtain a linearized equation in deviations in the canonical form:

$$T_{T_{g.2}D}\frac{d\left(\Delta T_{g.2}\right)}{dt}+\Delta T_{g.2}-K_{T_{c.2}/T_{g.2}D}\Delta T_{c.2}=K_{T_{g.1}/T_{g.2}D}\Delta T_{g.1}-T_{T_{g.2}D}\frac{d\left(\Delta T_{g.1}\right)}{dt}$$

Perform the Laplace transformation:

$$T_{T_{g.2}D}p\cdot T_{g.2}(p)+T_{g.2}(p)-K_{T_{c.2}/T_{g.2}D}T_{c.2}(p)=K_{T_{g.1}/T_{g.2}D}T_{g.1}(p)-T_{T_{g.2}D}p\cdot T_{g.1}(p)$$

Sum the similar terms and get the fourth equation in the variables:

$$\left(T_{T_{g.2}D}p+1\right)T_{g.2}(p)-K_{T_{c.2}/T_{g.2}D}T_{c.2}(p)=\left(K_{T_{g.1}/T_{g.2}D}-T_{T_{g.2}D}p\right)T_{g.1}(p) \tag{10}$$

We use Kramer's method to find the component transfer functions. Define numeric characteristics without taking into account heat loss to the environment. The matrix of system with columns $w_{c.2}$, $w_{g.2}$, $T_{c.2}$, $T_{g.2}$:

$$\Delta=\begin{bmatrix} T_{w_{c.2}}p+1 & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ 0 & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ 0 & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ 0 & 0 & -K_{T_{c.2}/T_{g.2}} & T_{T_{g.2}}p+1 \end{bmatrix}$$

Determinant of the system:

$$\left|\Delta\right|=\left(T_{w_{c.2}}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(\left(T_{T_{c.2}}p+1\right)\cdot\left(T_{T_{g.2}}p+1\right)\right)-\left(K_{T_{g.2}/T_{c.2}}\cdot K_{T_{c.2}/T_{g.2}}\right)\right)\right]$$

Transposed coefficients column vector by control $T_{g.1}$:

$$T_{g.1}=\left[-K_{T_{g.1}/w_{c.2}} \quad K_{T_{g.1}/w_{g.2}} \quad K_{G_{c.1}/T_{c.2}} \quad \left(K_{T_{g.1}/T_{g.2}}-T_{T_{g.2}}p\right)\right]^T$$

The algebraic additions matrix by control:

$$\Delta_{T_{g.1}}=\begin{bmatrix} -K_{T_{g.1}/w_{c.2}} & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ K_{T_{g.1}/w_{g.2}} & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ K_{G_{c.1}/T_{c.2}} & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ \left(K_{T_{g.1}/T_{g.2}}-T_{T_{g.2}}p\right) & 0 & -K_{T_{c.2}/T_{g.2}} & T_{T_{g.2}}p+1 \end{bmatrix}$$

Determinant of the system by control:

$$\begin{aligned}\left|\Delta_{T_{g.1}}\right|=&-\left(K_{T_{g.1}/T_{g.2}}-T_{T_{g.2}}p\right)\left[-\left(T_{w_{g.2}}p+1\right)\left(\left(K_{T_{c.2}/w_{c.2}}\cdot K_{T_{g.2}/T_{c.2}}\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)\right)\right]+\\ &+K_{T_{c.2}/T_{g.2}}\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{T_{g.1}/w_{c.2}}\cdot K_{T_{g.2}/T_{c.2}}\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]+\\ &+\left(T_{T_{g.2}}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(-K_{T_{g.1}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)-\left(-K_{T_{c.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]\end{aligned}$$

Transposed coefficients column vector by perturbation $G_{c.1}$:

$$G_{c.1}=\left[K_{G_{c.1}/w_{c.2}} \quad 0 \quad K_{G_{c.1}/T_{c.2}} \quad 0\right]^T$$

The algebraic additions matrix by perturbation:

$$\Delta_{G_{c.1}}=\begin{bmatrix} K_{G_{c.1}/w_{c.2}} & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ 0 & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ K_{G_{c.1}/T_{c.2}} & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ 0 & 0 & -K_{T_{c.2}/T_{g.2}} & T_{T_{g.2}}p+1 \end{bmatrix}$$

Determinant of the system by disturbance:

$$\left|\Delta_{G_{c.1}}\right| = K_{T_{c.2}/T_{g.2}}\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{G_{c.1}/w_{c.2}}\cdot\left(-K_{T_{g.2}/T_{c.2}}\right)\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]+$$
$$+\left(T_{T_{g.2}}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{G_{c.1}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)-\left(-K_{T_{c.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]$$

Define the same characteristics with taking into account heat loss to the environment. The matrix of system with columns $w_{c.2}$, $w_{g.2}$, $T_{c.2}$, $T_{g.2}$:

$$\Delta D = \begin{bmatrix} T_{w_{c.2}}p+1 & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ 0 & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ 0 & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ 0 & 0 & -K_{T_{c.2}/T_{g.2}D} & T_{T_{g.2}D}p+1 \end{bmatrix}$$

Determinant of the system:

$$\left|\Delta D\right| = \left(T_{w_{c.2}}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(\left(T_{T_{c.2}}p+1\right)\cdot\left(T_{T_{g.2}D}p+1\right)\right)-\left(K_{T_{g.2}/T_{c.2}}\cdot K_{T_{c.2}/T_{g.2}D}\right)\right)\right]$$

Transposed coefficients column vector by control $T_{g.1}$:

$$T_{g.1}D = \left[-K_{T_{g.1}/w_{c.2}} \quad K_{T_{g.1}/w_{g.2}} \quad K_{G_{c.1}/T_{c.2}} \quad \left(K_{T_{g.1}/T_{g.2}D} - T_{T_{g.2}D}p\right)\right]^T$$

The algebraic additions matrix by control:

$$\Delta_{T_{g.1}D} = \begin{bmatrix} -K_{T_{g.1}/w_{c.2}} & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ K_{T_{g.1}/w_{g.2}} & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ K_{G_{c.1}/T_{c.2}} & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ \left(K_{T_{g.1}/T_{g.2}D} - T_{T_{g.2}D}p\right) & 0 & -K_{T_{c.2}/T_{g.2}D} & T_{T_{g.2}D}p+1 \end{bmatrix}$$

Determinant of the system by control:

$$\left|\Delta_{T_{g.1}D}\right| = -\left(K_{T_{g.1}/T_{g.2}D} - T_{T_{g.2}D}p\right)\left[-\left(T_{w_{g.2}}p+1\right)\left(\left(K_{T_{c.2}/w_{c.2}}\cdot K_{T_{g.2}/T_{c.2}}\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)\right)\right]+$$
$$+K_{T_{c.2}/T_{g.2}D}\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{T_{g.1}/w_{c.2}}\cdot K_{T_{g.2}/T_{c.2}}\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]+$$
$$+\left(T_{T_{g.2}D}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(-K_{T_{g.1}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)-\left(-K_{T_{c.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]$$

Transposed coefficients column vector by perturbation $G_{c.1}$:

$$G_{c.1D} = \left[K_{G_{c.1}/w_{c.2}} \quad 0 \quad K_{G_{c.1}/T_{c.2}} \quad 0\right]^T$$

The algebraic additions matrix by perturbation:

$$\Delta_{G_{c.1}D} = \begin{bmatrix} K_{G_{c.1}/w_{c.2}} & 0 & -K_{T_{c.2}/w_{c.2}} & K_{T_{g.2}/w_{c.2}} \\ 0 & T_{w_{g.2}}p+1 & K_{T_{c.2}/w_{g.2}} & -K_{T_{g.2}/w_{g.2}} \\ K_{G_{c.1}/T_{c.2}} & 0 & T_{T_{c.2}}p+1 & -K_{T_{g.2}/T_{c.2}} \\ 0 & 0 & -K_{T_{c.2}/T_{g.2}D} & T_{T_{g.2}D}p+1 \end{bmatrix}$$

Determinant of the system by disturbance:

$$\left|\Delta_{G_{c.1}D}\right| = K_{T_{c.2}/T_{g.2}D}\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{G_{c.1}/w_{c.2}}\cdot\left(-K_{T_{g.2}/T_{c.2}}\right)\right)-\left(K_{T_{g.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]+$$
$$+\left(T_{T_{g.2}D}p+1\right)\left[\left(T_{w_{g.2}}p+1\right)\left(\left(K_{G_{c.1}/w_{c.2}}\cdot\left(T_{T_{c.2}}p+1\right)\right)-\left(-K_{T_{c.2}/w_{c.2}}\cdot K_{G_{c.1}/T_{c.2}}\right)\right)\right]$$

The obtained transfer functions by disturbance channels (*Dc*) and control channels (*Cc*) are summarized in Table 1.

Transient characteristics of the control object according to the disturbance and control channels are presented in figures 2 and 3. The characteristics are determined taking into account (red solid line) and without taking into account (blue dashed line) the assumptions about heat loss to the environment.

**Table 1 – Transfer functions**

| | with taking into account heat loss to the environment | without taking into account heat loss to the environment |
|---|---|---|
| control channels | $W_{Cc\mathrm{D}} = \frac{\lvert\Delta_{T_{g.1}D}\rvert}{\Delta D}$ | $W_{Cc} = \frac{\lvert\Delta_{T_{g.1}}\rvert}{\Delta}$ |
| disturbance channels | $W_{DcD} = \frac{\lvert\Delta_{G_{c.1}D}\rvert}{\Delta D}$ | $W_{Dc} = \frac{\lvert\Delta_{G_{c.1}}\rvert}{\Delta}$ |

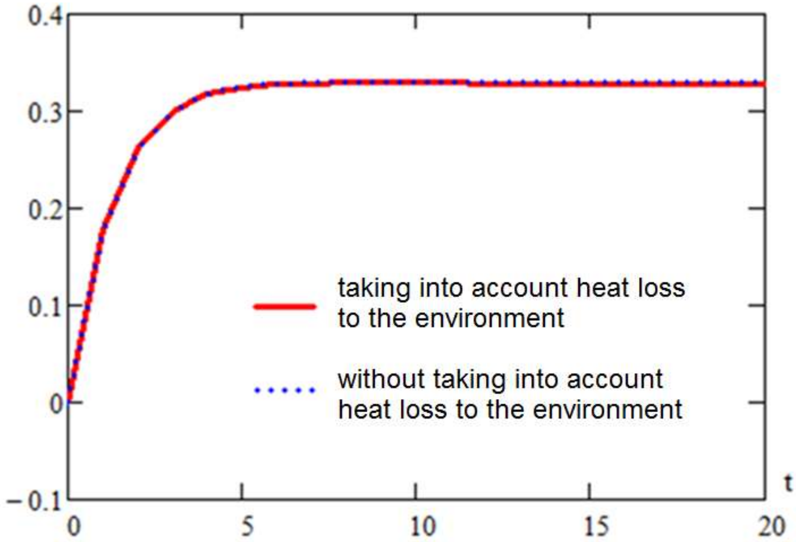


**Fig. 2 – Transient characteristics of the control object for the control channel $T_{g.1}$→$w_{c.2}$**

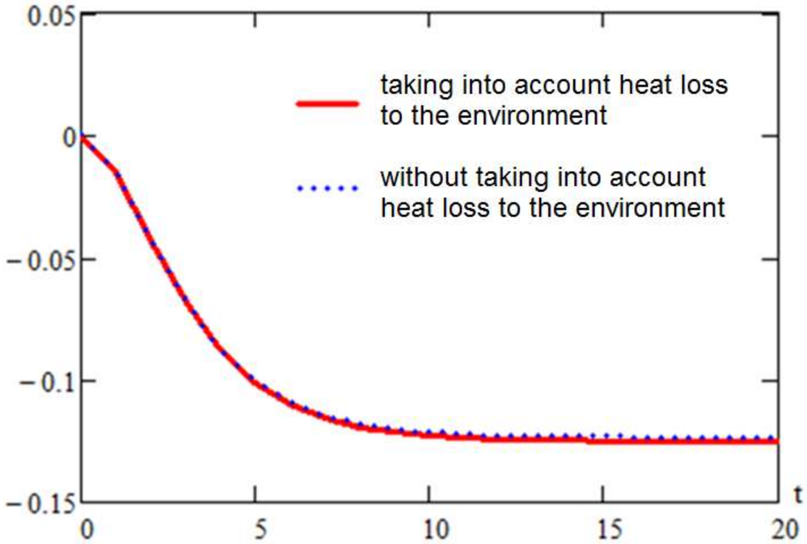


**Fig. 3 – Transient characteristics of the control object for the disturbance channel $G_{c.1}$→$w_{c.2}$**

**Conclusions.** The presented research results made it possible to develop and analyse a mathematical model of a spray dryer taking into account the requirements for the moisture content of the dried substance and the analysis of the control process by changing the drying agent temperature. The spray dryer's transient characteristics calculated and presented in the research by the disturbance and control channels made it possible to compare the control object behaviour both with and without taking into account the assumptions.

**Further research prospects.** The further research, the resulting mathematical model of the spray dryer would be used in a control system synthesis with different types of regulators. The control ensures only the entire system stability without any disturbances, but the adaptive system compensates the disturbances effects. In order to solve the issue of disturbances compensation and resource consumption optimization, it is necessary to develop the adaptive system structure for controlling the spray dryer operating mode.

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**РОЗРОБЛЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ РОЗПИЛЮВАЛЬНОЇ СУШАРКИ ДЛЯ ЗАДАЧ КЕРУВАННЯ**

*Парадигма сталого розвитку, яка стає провідною у сучасному світі, включає у себе максимально ефективне використання матеріальних та енергетичних ресурсів у виробничих процесах. Одним зі шляхів досягнення такої мети є створення ефективних автоматизованих систем керування. У той же час, ефективне функціонування таких систем неможливе без адекватних математичних моделей об'єктів керування. Таким чином, актуальним завданням даного дослідження є створення моделі об'єкту – розпилювальної сушарки, яку можна було б використовувати в системі керування.*

*Одним з основних апаратів у виробництві триполіфосфату натрію є розпилювальна сушарка, в якій водна суспензія солей осушується димовими газами. У роботі досліджено конвективне сушіння, коли матеріал безпосередньо контактує з сушильним агентом – пічним газом. Контролювати процес сушіння за температурою сушильної речовини. Під час розроблення математичних моделей прийнято ряд припущень, які дозволяються спростити математичний опис процесу. У той же час, математична модель роботі*

*розробляється виходячи з розділених умов – для випадку коли втратами тепла від розпилювальної сушарки у навколишнє середовище нехтуємо та для випадку, коли втрата тепла у зовнішнє середовище пропорційна різниці температур між топковим газом в сушарці і зовнішнім середовищем.*
*Початком дослідження є створення структурно-параметричної схеми розпилювальної сушарки з урахуванням усіх вхідних і вихідних величин, що дозволяє визначити канали збурень керування. Складається матеріальний баланс за кількістю вологи і сипучих речовин і тепловий баланс газового середовища. Оскільки метою розроблених моделей є їх використання для вирішення задач керування, у роботі виконано повний шлях перетворень моделей для одержання передатних функцій за каналами керування та збурення, а саме лінеаризація, перетворення за Лапласом, одержання канонічного виду тощо. Розрахунок компоненти передатних функцій здійснюється методом Крамера. На фінальному етапі дослідження одержано та порівняно перехідні характеристики за каналами керування та збурення для систем керування, які будуть працювати на основі розроблених моделей. Одержано чотири перехідні характеристики: за каналом керування з врахуванням припущення про втрату тепла у навколишнє середовище і без врахування, та за каналом збурення за тих самих припущень.*
*Наведені результати дослідження дають змогу побудувати та проаналізувати математичну модель розпилювальної сушарки з урахуванням вимог до вологості речовини, що висушується, та аналізу процесу керування зміною температури сушильного агенту. Перехідні характеристики розпилювальної сушарки за каналами збурення та керування, які розраховані та представлені в дослідженні, дають змогу порівняти поведінку об'єкта керування з урахуванням і без втрат тепла в навколишнє середовище.*

***Ключові слова:** енергоефективність, розпилювальна сушарка, математична модель, канонічний вигляд, лінеаризоване рівняння.*

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