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METHOD FOR PID-TUNING VIA FEEDBACK CONTROL SYSTEM POLE PLACEMENT

Pole placement is the only PID-tuning technic that allows one to obtain a control system with desired, and, moreover, highly predictable performance and control quality. Number of controller tuning parameters is equal to number of poles closed-loop poles it can precicely place, so that PID-controller can place exactly three poles, and PI- can place only two. For this reason PI-controller is best used with first-order processes (second-order closed loop system), and PID-controller with second-orded ones (third-order closed loop system). However, many processes have higher order than two, and still are controlled with PID-controllers. To tune it using pole placement techniques, it is necessary to consider only dominant poles, which affect performance of the system to the greatest extent. First, it is necessary to study a PI-controller with a second-order process, which is the most basic case. Tuning is performed using global optimization methods to fit dominant poles of a tuned system to dominant poles of a reference system.

Keywords: *PID-tuning, pole placement, optimization, PI-controller, second-order system*

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The Problem Statement. PID-controllers are the most widely used type of controller in industrial plants control systems due to their simplicity and good performance. Many different approaches for tuning of PID-controller, i.e., properly choosing controller parameters, are theoretically studied. Many of them are empirical and heuristic, like Ziegler-Nichols tuning rules and their numerous extensions, or frequency response shaping. However, the only way that might guarantee desired performance of the system is pole placement. If all the poles have rather large negative real part, the rise time of the system is then bounded by a desired value, and if in addition to that imaginary parts of complex-conjugate poles are rather small, the damping ratio and settling time will be close to desired.

There are many ways to optimize system behaviour, including integrated criteria like integrated error (IE), integrated squared error (ISE) and more complicated functions. On the other hand, it is possible to fit poles directly, using weighted sum of squared distances between corresponding poles of reference and real systems:

$$\Im = \sum_{i=1}^{n} \left(k_i^{Re}\left(Re(s_i) - Re(s_i^r)\right)^2 + k_i^{Im}\left(Im(s_i) - Im(s_i^r)\right)^2 \right) \to min \qquad (1)$$

where $s_j s_j$ is j-th pole of the real system, $s_j^r s_j^r$ is j-th pole of the reference system, $Re Re$ and $Im Im$ stands for real part and imaginary part of a complex number respectively, $k_j^{Re} k_j^{Re}$ and $k_j^{Im} k_j^{Im}$ are nonnegative real weighing coefficients, $n n$ is the number of poles of the system. This criterium might be turned into zero if all poles are fitted perfectly, of just minimized if some poles are impossible to fit simultaneously.

Benefits of fitting poles directly are mainly connected with specific objectives which may be set for the system, e.g., smaller raising time might be preferred in spite of smaller damping ratio or larger overshoot, which cause ISE criterion to indicate worse results in that case.

Analysis of previous research. Problem of PI- and PID-controller synthesis remains relevant for many decades. Results in the field are diverse.

Description of wide variety of conventional methods and system structures for PID-control may be found in book [1].

In paper [2] a coefficient-fitting approach is examined both for continuous and discrete-time closed-loop systems with PID-controller. Fitting is performed for overdetermined equation systems for coefficients of a reference and tuned systems, modified to guarantee response stability. The least-squares method was utilized for optimal solution search. However, results may be not optimal in sense of system poles closeness to reference system poles.

In paper [3] a generalized PID-controller was synthesized for a control system with two feedback loops, outer and a nested one. The criterium for optimization was minimum of integrated squared response plus weighted squared control signal in case of system response to delta impulse input. Although systems with such structure have good performance, such controller includes several integrators and differentiators and is considerable harder to implement, and might not be necessary for industrial processes.

The approach with generalized PID-controller is further developed in research [4], where problem is posed as optimal open-loop zero placement. Like in other works of these authors, impulse response tracking was applied for tuning. In [5] closed-loop pole-placement is chosen instead of open-loop zero placement. In addition, a prefilter is utilized. In all papers [2 – 5] much analytical computation is performed before applying numeric optimization, including some consideration of state-controllers and state-observers, which complicates analysis.

Nowadays, optimization-based techniques for PID-tuning are the object of active research.

An extensive theoretical study of least-squares optimal pole placement for linear time-invariant systems in state-space representation was done in [6]. As optimality criterium a matrix function was proposed. Some theorems were proved on global optimum existence and convergence of optimization. However, no physical meaning of such functions was introduced, and the dynamics of presented optimal systems also was out of scope.

The idea of optimal dominant pole placement using plain PID-controller was compared with classical Ziegler-Nichols tuning rules in case of system with time delay in article [7]. The criterium was integrated absolute error minimum, and the results were far better than obtained using Ziegler-Nichols method.

For a process, for which no mathematical model was developed, an iterative step response shaping method was proposed in [8].

Analytical eigenvalues placement methods for a process with known state space representation are also developed. For example, in [9] one specific method utilizing Moore's algorithm for optimal system eigenvalues placement is described and compared to its alternatives. The optimization objective was to minimize system sensitivity to perturbations, so that closed-loop response was both swift and robust.

The purpose of this article is to develop an implementation of optimal pole placement PID-controller tuning and to test it on several processes. Results should be examined for qualitative differences for different types of processes.

Presentation of the main material.

In the current paper, only second-order processes are considered, so that the process transfer function is

$$W_p(s) = \frac{K}{(s+\lambda_1)(s+\lambda_2)} \tag{2}$$

where KK is static gain, $-\lambda_1 - \lambda_1$ and $-\lambda_2 - \lambda_2$ are stable poles, either real or complex-conjugate. PID-controller transfer function has form

$$W_c = P + \frac{I}{s} + Ds = \frac{Ds^2 + Ps + I}{s} \tag{3}$$

where PP, II and DD are proportional, integral and derivative gains respectively. Closed loop system has transfer function

$$W = \frac{W_c W_p}{1 + W_c W_p} = \frac{KD \cdot \left(s^2 + \frac{P}{D}s + \frac{I}{D}\right)}{s^3 + (\lambda_1 + \lambda_2 + D)\cdot s^2 + (\lambda_1\lambda_2 + P) + I} \tag{4}$$

Thus, the characteristic polynomial of the system is

$$s^3 + (\lambda_1 + \lambda_2 + D)\cdot s^2 + (\lambda_1\lambda_2 + P) + I = (s+s_1)(s+s_2)(s+s_3) \tag{5}$$

Roots of such a polynomial are fully determined by is coefficients, so that using PID-controller allows one to fit all poles of the system perfectly. In addition to that, a closed-loop system will always be minimal phase if only all PID-controller parameters are positive, which is fairly easy to guarantee. This implies that using optimal poles placement for PID and second-order system must result in an equality of corresponding poles. However, for PI-controller the second coefficient of a polynomial may not be affected by tuning, so that only two poles might be tuned. Below, we consider that $s_1 \le s_2 \le s_3 s_1 \le s_2 \le s_3$, i.e., $-s_1 - s_1$ is the dominant pole.

It is expected, that PID-tuning for second-order process is weakly dependent on weighing coefficients in objective function. Unlikely behaves system with PI-controller. As far as PI-controller cannot place all three poles perfectly, including non-dominating pole in the objective function is likely to make results even worse. For this reason, some roots must be excluded. For third-order system there are three possible relative poles placement: three real poles, dominating real pole, dominating complex-conjugate poles. This might affect performance of the method with different weighing coefficients, so that we examine different combinations to compare results.

Now let us take a closer look on the algorithm itself. It is well known, that roots of a polynomial are unstable function of its coefficients, i.e., small changes in polynomial coefficients may result in a dramatically changed roots, including both numeric values and structure (number of real and complex-conjugate roots etc.). Due to this, zero-order numeric optimization method must be used to avoid computational problems in points where derivative of the objective function is undefined or infinite. To set up correspondence between two sets of poles, we used sorting in descending order by real parts of the roots. To avoid getting stuck next to local minima, we use local optimization with multiple starting points, uniformly distributed in some square or cubic segment of two- or three-dimensional space of tuning parameters respectively. To make code more flexible, pole exclusion is done using zero weighing coefficients in objective function. Optimization is performed using a Matlab routine. Code is placed in the cloud and may be accessed via URL https://drive.google.com/drive/folders/1hYkY3Xcjxb8tISRcEfRZnoNksqKp-DsC?usp=sharing.

Six different processes were examined: three aperiodic (with real poles) and three oscillatory (with complex-conjugate poles):

$$W_{p1} = \frac{1}{(s+0{,}4)^2} \quad (6)$$

$$W_{p2} = \frac{1}{(s+0{,}1)(s+0{,}5)} \quad (7)$$

$$W_{p3} = \frac{1}{(s+0{,}01)(s+0{,}2)} \quad (8)$$

$$W_{p4} = \frac{1}{(s+0{,}4-0{,}2j)(s+0{,}4+0{,}2j)} \quad (9)$$

$$W_{p5} = \frac{1}{(s+0{,}1-0{,}1j)(s+0{,}1+0{,}1j)} \quad (10)$$

$$W_{p6} = \frac{1}{(s+0{,}01-0{,}3j)(s+0{,}01+0{,}3j)} \quad (11)$$

Nine sets of desired poles were set, three for each type of pole set. To define a reference system, we calculated a PID-controller for each process and reference poles combination. Such PID-controller is aimed to fit all three poles of the reference systems. The key difference of such implementation of a reference system is existence of two zeros, which also impact dynamics of the system.

Table 1 – Poles of a reference closed-loop system

Number	Pole set type	Pole values		
1	Complex-conjugate dominating poles	-0,2 - 0,2j	-0,2 + 0,2j	-1
2		-0,5 - 0,1j	-0,5 + 0,1j	-0,7
3		-0,7 - 0,3j	-0,7 + 0,3j	-1,1
4	Real dominating pole	-0,25	-0,6 - 0,1j	-0,6 + 0,1j
5		-0,45	-0,7 - 0,2j	-0,7 + 0,2j
6		-0,5	-0,1 - 0,3j	-0,1 + 0,3j
7	Three real poles	-0,3	-0,4	-0,8
8		-0,5	-0,8	-1
9		-0,2	-0,4	-0,6

The following four different sets of weighing coefficients were defined for optimization. First set takes into account only dominating pole (both real and imaginary part), so does the second set but for two right-most poles. The third takes into account all poles equalizing their importance. In the fourth set, dominating pole has larger coefficient for both real and imaginary part.

Table 2 – Weighing coefficients sets

Coefficient set number	k_1^{Re}	k_1^{Im}	k_2^{Re}	k_2^{Im}	k_3^{Re}	k_3^{Im}
1	1	1	0	0	0	0
2	1	1	1	1	0	0
3	1	1	1	1	1	1
4	4	4	1	1	1	1

Some results of data visualization are presented on the figures below. Main attention is paid to comparison between tuning quality for different objects, different controllers and weighing types. Pole placement on the complex plane is shown for reference and real systems, surface and volume plots for objective function, step and impulse response plots. Complete set of generated data may be accessed via URL https://drive.google.com/drive/folders/1MFlm7Zd7Y6KNpcfkolOlEGERzqcQy2wM?usp=sharing.

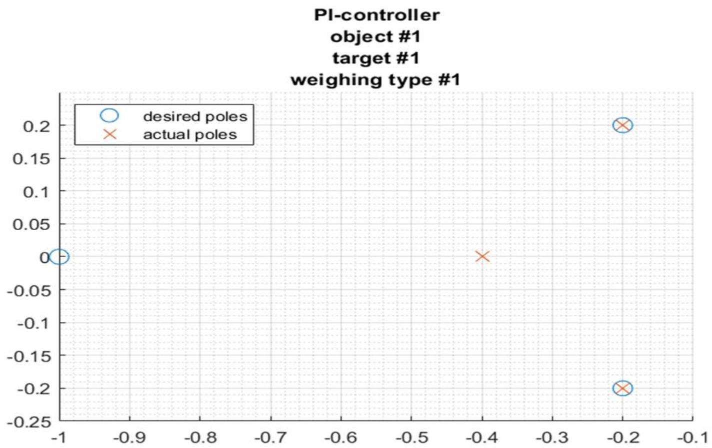

Fig. 1 – Pole placement, aperiodic process p1 with PI-controller, target pole set 1, weighing coefficients 1

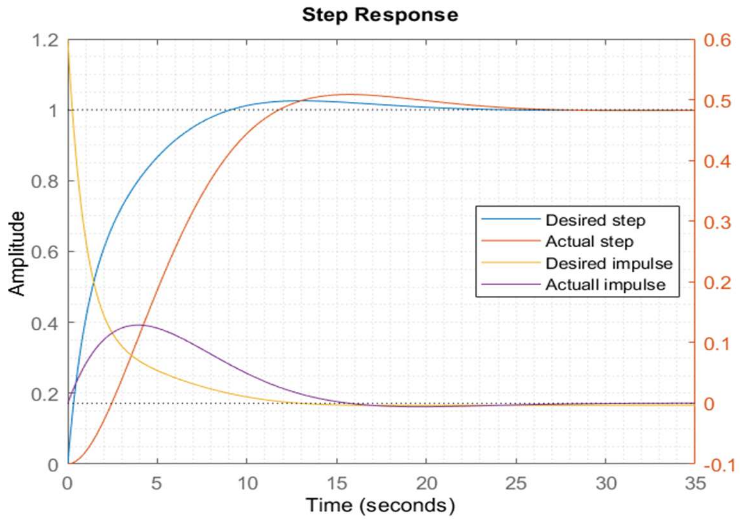

Fig. 2 – Step and impulse response, aperiodic process p1 with PI-controller, target pole set 1, weighing coefficients 1

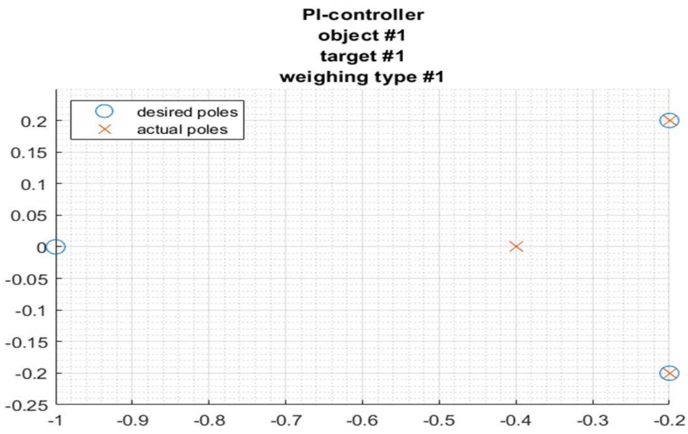

Fig. 3 – Pole placement, oscillatory process p4 with PI-controller, target pole set 1, weighing coefficients 1

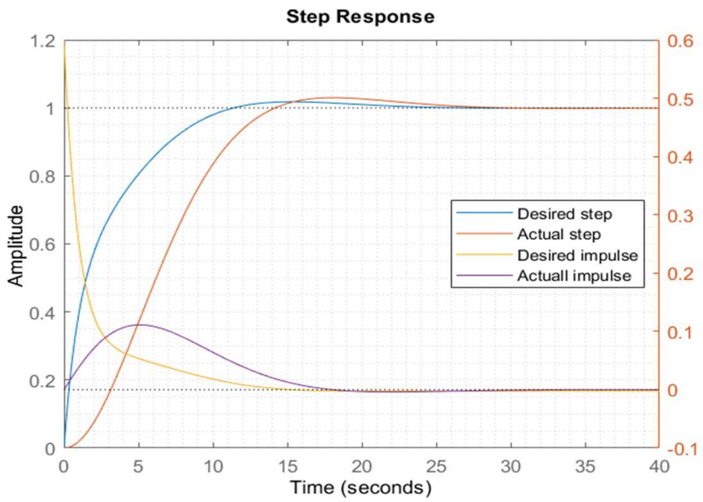

Fig. 4 – Step and impulse response, oscillatory process p4 with PI-controller, target pole set 1, weighing coefficients 1

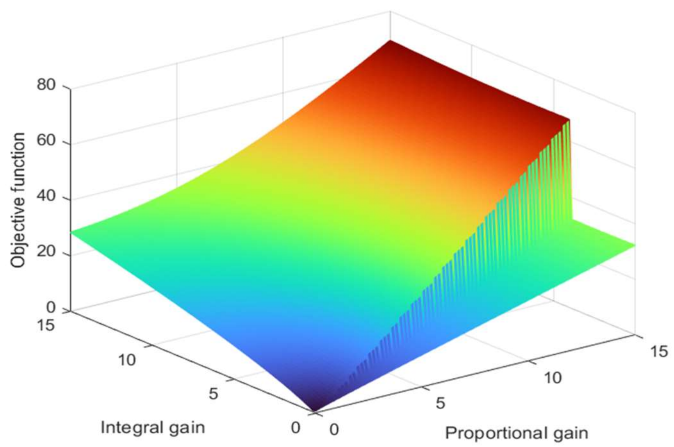

Fig. 5 – Objective function surface plot, corresponding to fig. 1 and fig. 2

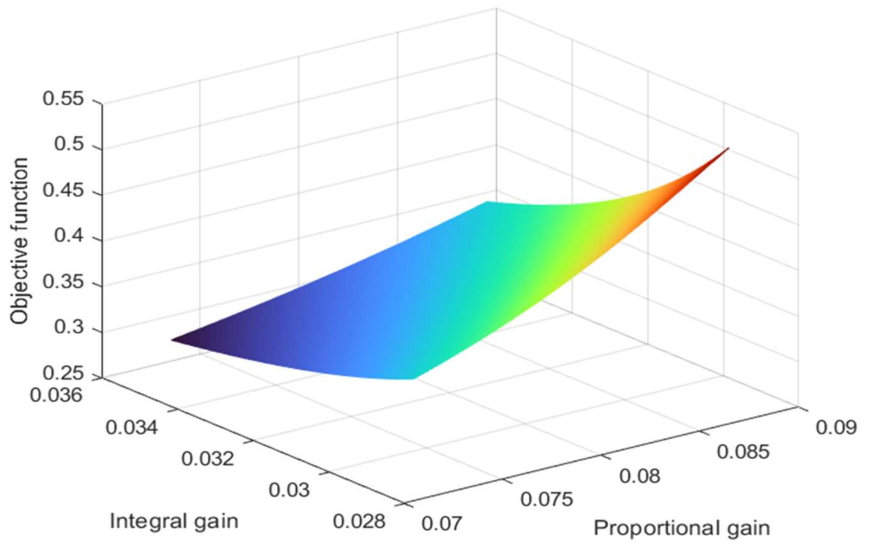

Fig. 6 – Objective function surface plot in small neighborhood of the global optimum, aperiodic process p1 with PI-controller, target pole set 1, weighing coefficients 1

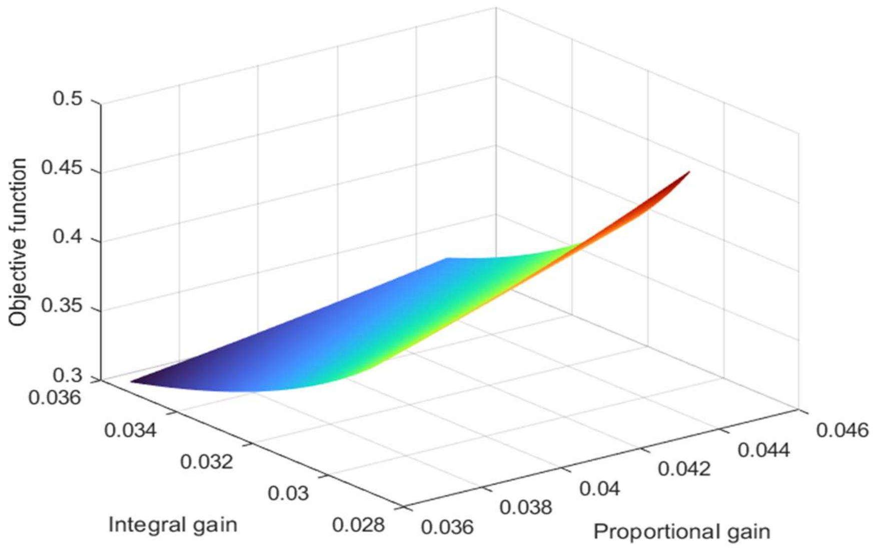

Fig. 7 – Objective function surface plot in small neighborhood of the global optimum, oscillatory process p4 with PI-controller, target pole set 1, weighing coefficients 1

As it can be seen from figures, poles for both systems are equal for naked eye, but step responses indicate a small difference between them for processes p1 and p4. It may be explained by computation error. Surface plot on fig. 5 is typical for PI-controller tuning problem solved here, and is varying mostly in a close neighborhood of the global optimum. That means, that all optimal PI-controller tunings have rather small proportional and integral gains, regardless of existence of single or multiple local minima. For two examples given above, the appearance of the surface is mostly the same in qualitative sense, still objective function values are slightly different, and tuning parameters, exactly proportional gain, are significantly different. In the same time, not all processes may be tuned for fitting this dominating poles pair so well.

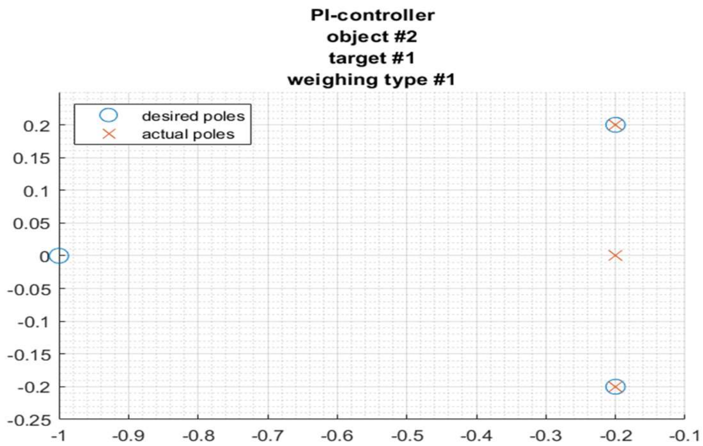

Fig. 8 – **Pole placement, aperiodic process p2 with PI-controller, target pole set 1, weighing coefficients 1**

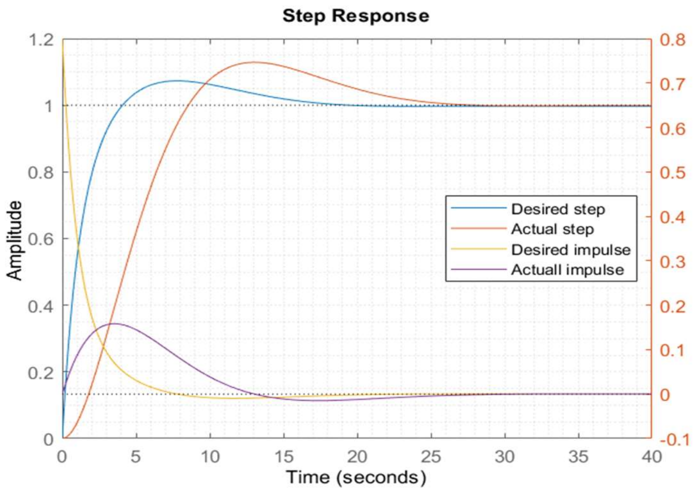

Fig. 9 – **Step and impulse response, aperiodic process p2 with PI-controller, target pole set 1, weighing coefficients 1**

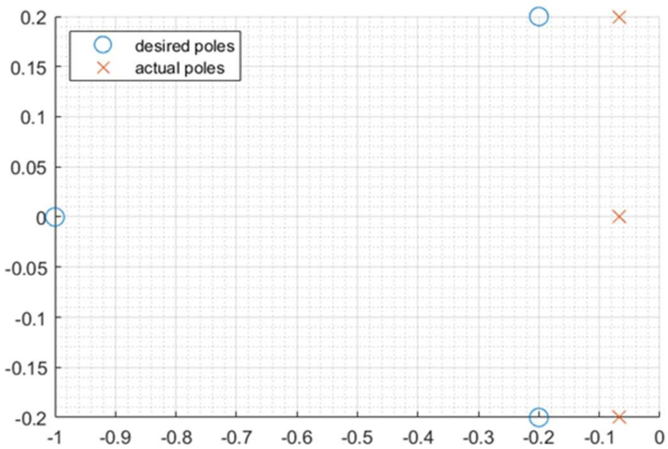

Fig. 10 – **Pole placement, oscillatory process p4 with PI-controller, target pole set 1, weighing coefficients 1**

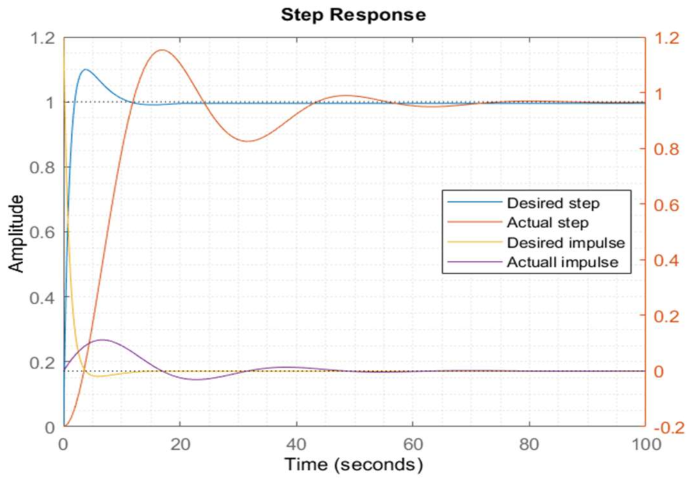

Fig. 11 – Step and impulse response, oscillatory process p4 with PI-controller, target pole set 1, weighing coefficients 1

Process p2 was tuned in the way the system has three poles with equal real parts, which is a boundary situation when further moving of poles to the left is no more possible. Such relative location of poles is actually the most common in our results for PI-controllers. A similar location is shown on fig. 10 for process p4. Unlike tuning for p2 where dominating real part is fitted perfectly, oscillatory object could not reach such pole placement and got stuck on poles with real part slightly above -0,1. However, imaginary parts of both pole sets are the same. Fig. 9 and fig. 11 illustrates how much better is step response of system with aperiodic process than with an oscillatory one.

Not only imaginary parts affect the tunability of the system, but also position of recessive poles of the process itself. Consider fig. 12 to fig. 15 with tuning results for process p1 and pole set 2 and weighing coefficient sets 1, 2, 3 and 4 respectively. The worst result corresponds to using all three poles with the same weights. This causes dominating poles of the real system to be located to the right from their possible location, where they take place for weighing coefficient sets 2 and 4. For all systems with weighing coefficient set 3 a similar result is reached: poles are placed like in the reference system, but shifted to the right. Sometimes this shift produces unstable poles. For this reason, we consider equal poles weighing much less useful.

Comparing results for weights 1, 2 and 4, we found out that, surprisingly, complex-conjugate poles for weights 4 have the smallest value of imaginary part, that is approximately equal to imaginary part of reference poles. For weights 1 and 2 imaginary part is deviating from the desired value for an unknown reason, especially for weights 1. Simultaneously, we can conclude that real part of system poles for process 1 with PI-controller is located in the segment they are located. Such limitation is observed for all analyzed systems with different real parts.

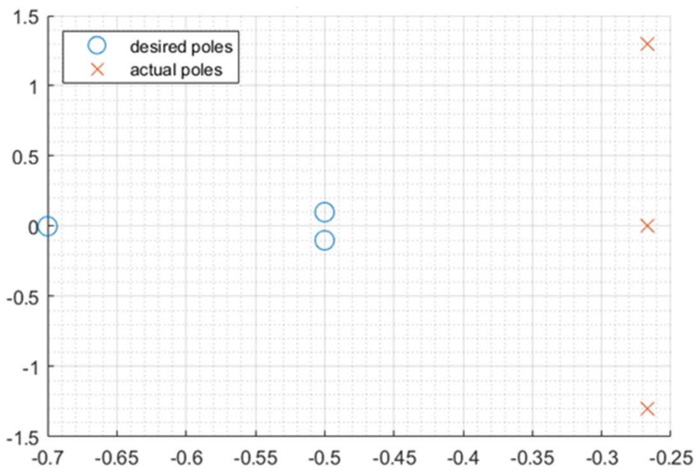

Fig. 12 – Pole placement, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 1

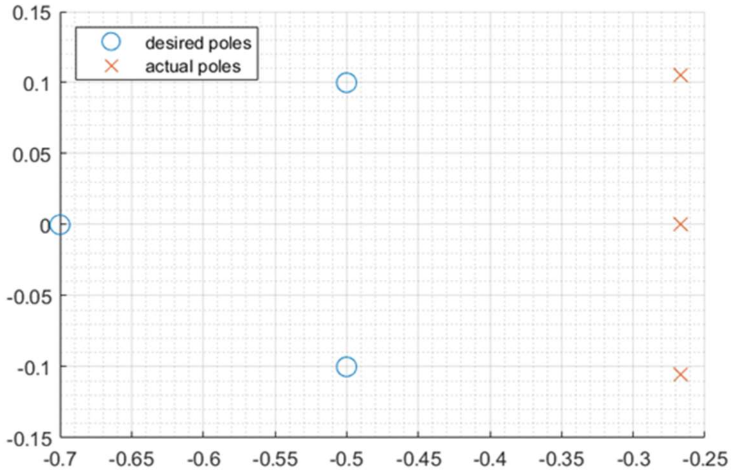

Fig. 13 – Pole placement, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 2

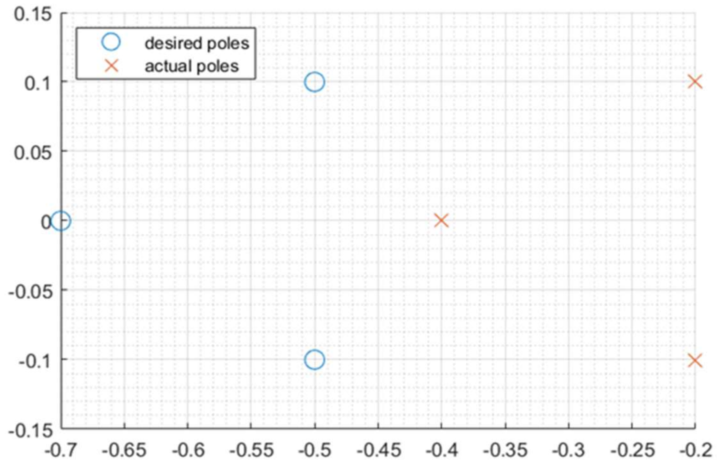

Fig. 14 – Pole placement, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 3

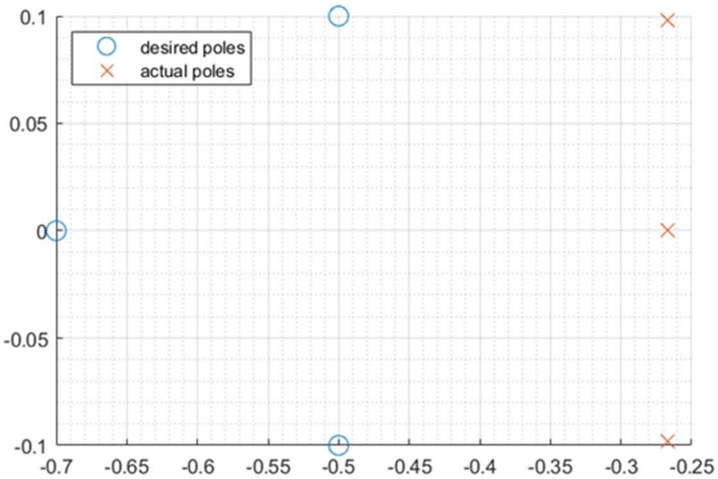

Fig. 15 – Pole placement, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 4

Also, we consider that found tuning limit for p1 has greater real part, than real part of initial system poles (taking the sign into account). On the other hand, limiting real part for process p2 has smaller real part, than real part of the dominating pole. This led us to the statement, that location recessive poles of the process are important if a controller of low order is to be tuned. Smaller real parts of recessive poles give more flexibility for closed system dominant pole fitting. These four tuning problems are also different in sense of objective function in the neighborhood of the local optimum. Figures from 16 to 19 illustrate this. It means, that for some reason optimization problems for weighing coefficient sets x and y are similar, but rather different for z and w.

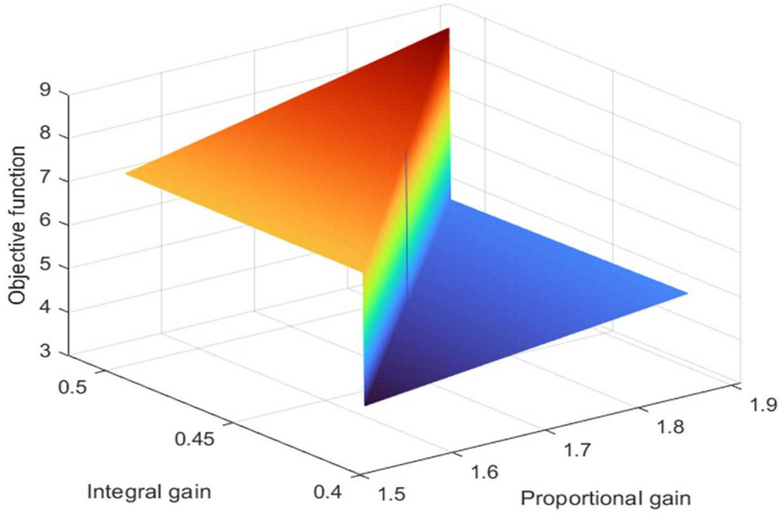

Fig. 16 – Objective function surface plot in small neighborhood of the global optimum, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 1

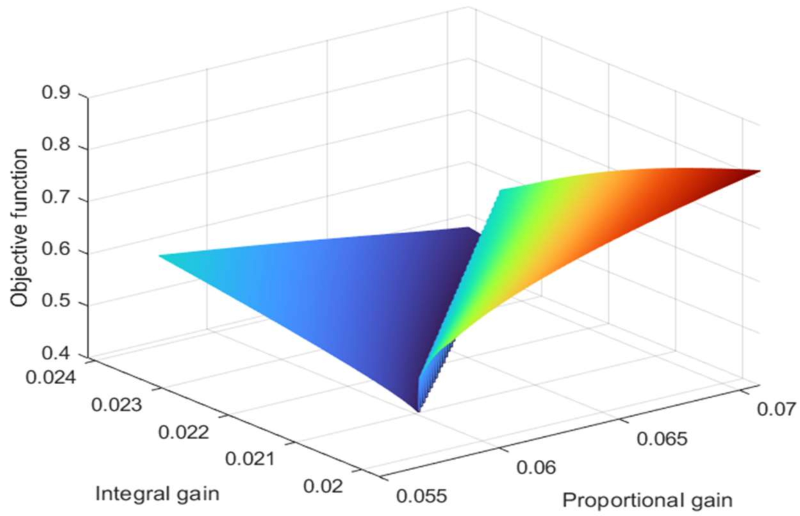

Fig. 17 – Objective function surface plot in small neighborhood of the global optimum, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 2

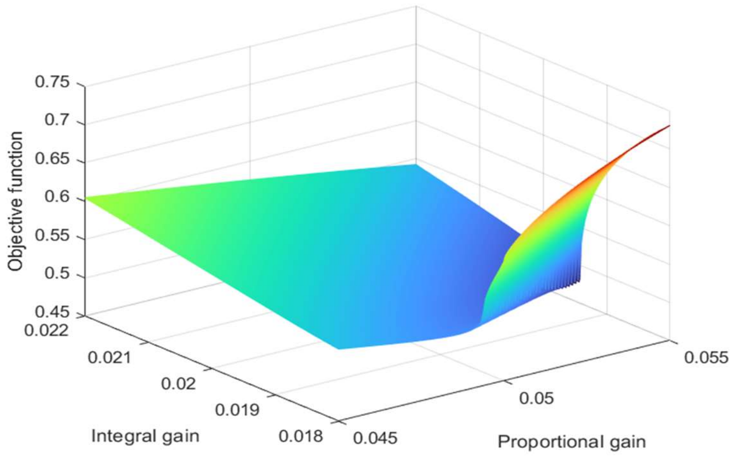

Fig. 18 – Objective function surface plot in small neighborhood of the global optimum, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 3

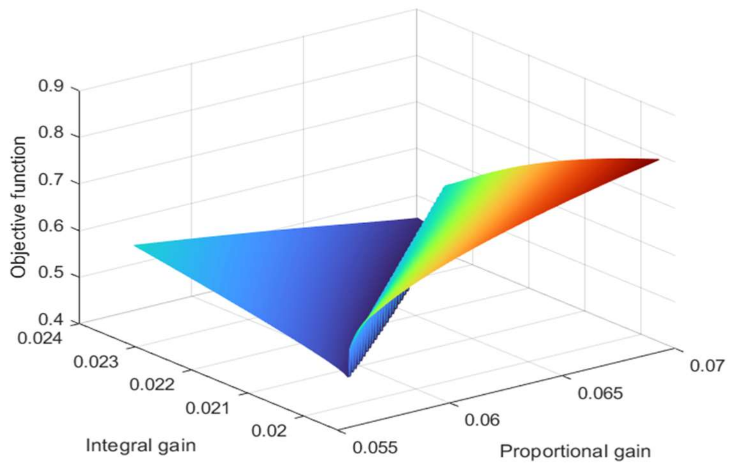

Fig. 19 – Objective function surface plot in small neighborhood of the global optimum, aperiodic process p1 with PI-controller, target pole set 2, weighing coefficients 4

Also, pole placement was applied to systems with PID-controller in order to test this method for tuning a controller with unlimited possibilities and limited information about reference system poles. Consider fig. 20 and fig. 21. As it was expected, both weighing coefficient sets are suitable to fit the real dominant pole. Although weights 2 were also supposed to fit both recessive poles, they did not. To examine optimization segment on multiple local minima, the volume plot for objective function was created (fig. 22 and fig. 23) for two different neighborhoods of the found optimum. However, nothing was found by eye, so we are unable to explain such behaviour now. It is probable that a pit both around the local optimum and around global optimum is rather shallow so that it is impossible to see on the plot. Similar nonconformity had arisen also for reference system 5 with all processes. Although tuned systems must have better performance, this is not guaranteed without fitting all poles which were meant to be fit.

For all other systems weights 3 and 4 resulted in perfect poles coincidence, as it was expected.

It is worth noting, that volume plot for objective function depicted on fig. 22 is as typical for PID-controller tuning, as surface from fig. 5 is typical for PI-controller tuning problems. Plots in data repository seems to be different because plot shown here was rotated for better visibility of its curly structure in the neighborhood of the extremum. Plots on smaller scales differs as well as surfaces for PI-controller do.

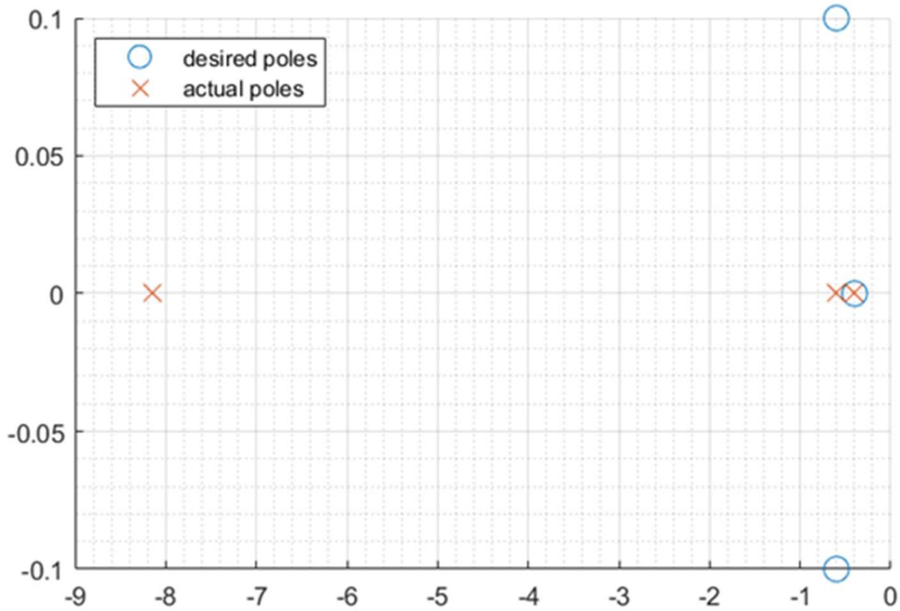

Fig. 20 – Pole placement, oscillatory process p4 with PID-controller, target pole set 4, weighing coefficients 1

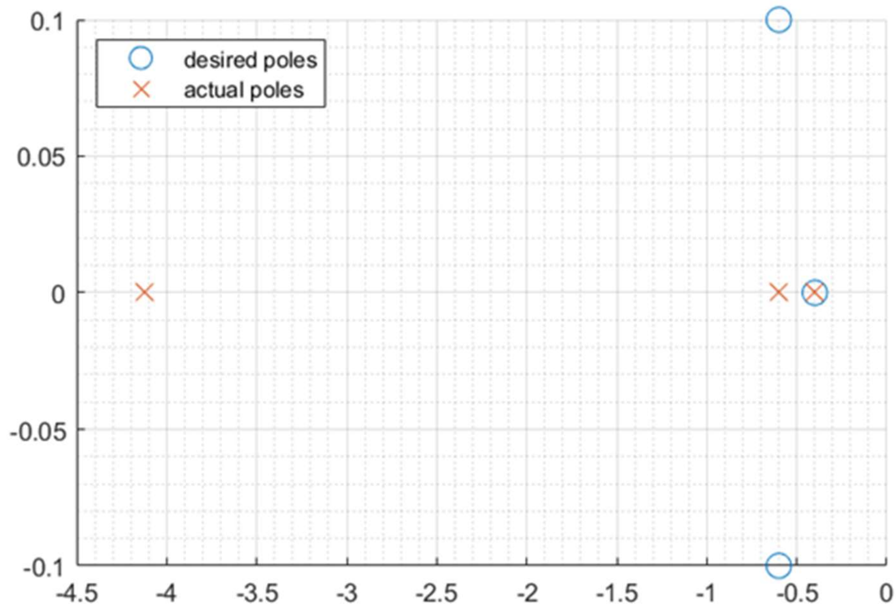

Fig. 21 – Pole placement, oscillatory process p4 with PID-controller, target pole set 4, weighing coefficients 2

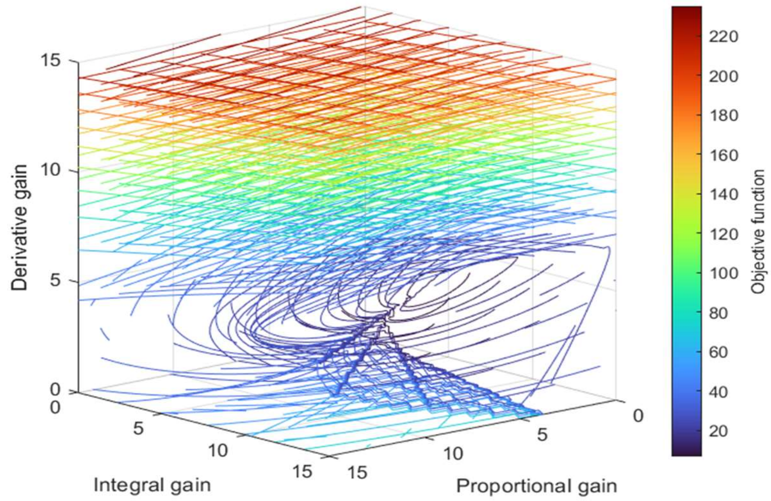

Fig. 22 – **Objective function volume plot for system from fig. 17 in the entire optimization space segment**

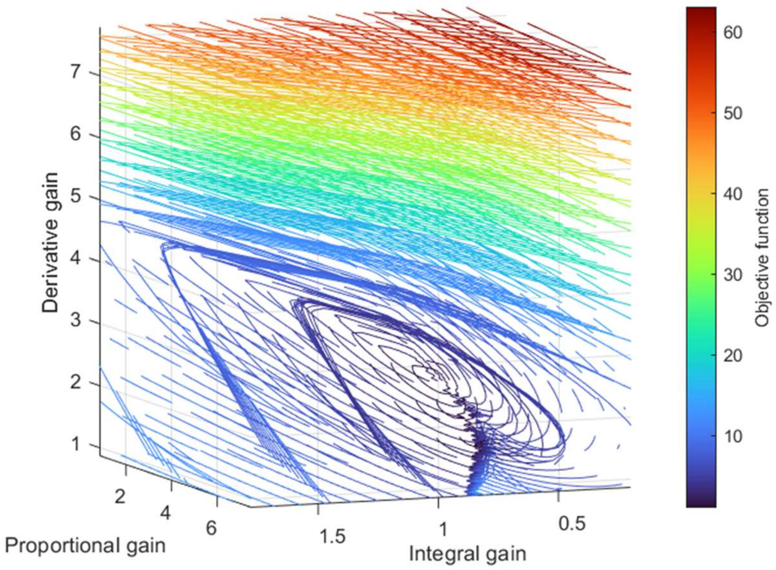

Fig. 23 – **Objective function volume plot for system from fig. 17 in the close neighborhood of the optimum**

Conclusions. A new method for pole-placement PID-tuning was proposed. For a system with PID-controller and a second-order process, it is capable of reaching full coincidence of references and real system poles. For some reason, this objective was not reached using dominating and one of complex-conjugate recessive poles

For PI-controller tuning, it allows to fit a dominant pole or a dominant pair of complex-conjugate poles with satisfying performance of the resulting system. However, if reference poles are too fast (located too far from the imaginary axis), it may be impossible to tune a PI-controller to fit even a single pole with the corresponding desired one. In this case, all three poles of tuned system have the same real part, but imaginary part of complex-conjugate poles may vary inside rather wide bounds. The best fit of imaginary part of such poles is reached, if all poles of the reference system have non-zero weighing coefficients, but dominant poles have larger weight. It was empirically shown that recessive pole of an aperiodic process can significantly affect the limitation imposed on real part of closed-loop system poles. The smaller is real part of the recessive pole, the better tunability system has.

The worst performance of the method for PI-controller is observed in case of counting all reference system poles with equal coefficients. For this weighing type, relative location of tuned system poles is similar to relative location of reference system poles, but all tuned poles are shifted to the right. This results in bad system performance with large raising time, or even in system instability due to a pole in the right half-plane arisen.

Objective function for both PI- and PID-controller tuning problem was visualized. All plots for PI-tuning problems are similar on large scales, but differ on small scales. Volume plots for PID-controllers look alike. These plots play rather illustrative role, still they might turn out to be theoretically important in the future.

Prospects of further research. In future research, the real-part-limit for PI-controller tuning for second order systems should be analyzed depending on process parameters. Analogic work must be done for high-order systems for tuning both PI- and PID-controllers. For an arbitrary structure of reference system poles, it is necessary to derive the best coefficient set, which will result in a system with dynamics as close to the desired as possible. The latter problem appears to be an optimization problem itself, so that optimization methods might be applied to it. In the current study, margins for optimization segment of the tuning parameter space were chosen arbitrarily. For more complex systems with different coefficients, is necessary to develop method for choosing a segment of minimal size, so that number of starting points for optimization could be decreased to minimum, resulting in faster computations.

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МЕТОД НАЛАШТУВАННЯ ПІД-РЕГУЛЯТОРА ЗА ЗАДАНИМ РОЗТАШУВАННЯМ ПОЛЮСІВ СИСТЕМИ КЕРУВАННЯ

Розміщення полюсів замкненої системи гарантує отримання потрібних динамічних процесів. Кількість змінних параметрів регулятора визначає кількість полюсів, які з його використанням можливо розмістити. Значна кількість промислових процесів, які описані моделями порядку, вище другого, керуються ПІД-регуляторами. Через це потрібно використовувати методи розміщення домінантних полюсів замкненої системи. Це складно зробити аналітично, тому доцільно використовувати числову оптимізацію.

Система автоматичного керування представлена об'єктом другого порядку з відомою передавальною функцією й регулятором із невідомими параметрами. Всього досліджено шість об'єктів як із ПІ-, так і з ПІД-регулятором. Для налаштування використано безумовну багатовимірну оптимізацію. Задачею оптимізації є мінімізації зваженої суми відстаней між відповідними полюсами налаштовуваної та бажаної систем. З використанням числових методів середовища Matlab було побудовано графіки цільової функції для ПІ-регулятора (поверхня) й ПІД-регулятора (набір ліній рівня, що заповнюють тривимірний простір).

Налаштування системи з ПІД-регулятором у більшості випадків дозволяє точно відтворити задані полюси. Винятки становлять випадки з налаштування за парою комплексно-спряжених домінантних полюсів для випадку, коли недомінантний полюс було проігноровано.
На противагу ПІД-регулятору, використання ПІ-регулятора накладає помітні обмеження на множину досяжних полюсів замкненої системи. Для будь-якого об'єкту існує нижня межа дійсної частини домінантних полюсів, тобто існує вертикальна пряма, ліворуч від якої полюси системи з ПІ-регулятором не можуть знаходитись. При досягненні цієї межі всі три полюси системи знаходяться на одній вертикальній прямій, тобто мають однакову дійсну частину. Порогове значення дійсної частини залежить від полюсів об'єкту. Наприклад, для аперіодичного об'єкту з кратним полюсом -0,4 ця межа має приблизне значення -0,27, а для подібного коливального об'єкту з полюсами -0,4-0,2j і -0,4+0,2j ця межа близька до -0,8.
Результат налаштування залежить від вагових коефіцієнтів у цільовій функції. Врахування недомінантного полюсу з таким самим ваговим коефіцієнтом, що й для домінантних полюсів, при налаштуванні ПІ-регулятора призводить до суттєвого погіршення результатів. Врахування недомінантного полюсу зі меншим ваговим коефіцієнтом може дати кращі результати, ніж налаштування лише за домінантними полюсами, а саме набір полюсів із тими самим дійсними частинами, але з меншими уявними.
Графіки-поверхні цільової функції для ПІ-регулятора з будь-якого поєднання об'єкту, полюсів бажаної системи й вагових коефіцієнтів цільової функції на масштабах всього проміжку оптимізації мають подібний вигляд із глобальним мінімумом поблизу початку координат. На менших масштабах вигляд цих графіків може суттєво відрізнятися залежно від розташування полюсів об'єкту. Графіки можуть мати розриви, злами (лінії, на яких розрив має похідна цільової функції). Проте для аперіодичних і коливальних об'єктів, домінантні полюси яких мають однакові дійсні частини, цільова функція в близькому околі оптимуму має подібні за зовнішнім виглядом графіки попри те, що значення параметрів ПІ-регулятора помітно відрізняються.
Візуалізація тривимірного поля значень цільової функції для систем із ПІД-регулятором показала так само показала схожі між собою результати на великих масштабах, але зі значними відмінностями в близькому околі оптимуму. Графіки для аперіодичних та коливальних об'єктів із однаковими дійсними частинами домінантних полюсів є подібними.
Розглянутий метод налаштування регуляторів низького порядку дозволяє досягати розташування полюсів системи із заданим типом регулятора, яке є близьким до найкращого з усіх теоретично можливих.

***Ключові слова**: налаштування ПІД-регулятора, розташування полюсів, оптимізація, ПІ-регулятор, система другого порядку*

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